LIFE INSURANCE PORTFOLIO MANAGEMENT

A Dissertation

Presented to

the Faculty of the Department of Economics

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

by

John D. Stowe

December 1974

ACKNOWLEDGMENTS

This paper substantially benefited from the guidance and constructive criticism of the members of my dissertation committee, for which I am indebted. I am also grateful for the support and patience of my wife, Adette.

LIFE INSURANCE

PORTFOLIO MANAGEMENT

An Abstract of a Dissertation

Presented to

the Faculty of the Department of Economics

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

---

by

John D. Stowe

December 1974

ABSTRACT

Based on the institutional and economic environment in which life insurance companies operate, this study develops a theoretical portfolio model with sufficient empirical content to yield hypotheses about life insurance portfolio behavior which are readily tested with appropriate econometric techniques.

State quantitative and qualitative restrictions on portfolio composition, the accounting procedures promulgated by the National Association of Insurance Commissioners, Federal tax laws, and other taxes and regulations provide the institutional setting for the life insurance investment process. Given these, the solvency of a life insurance company is affected by the riskiness of its investment portfolio, the uncertainty of future operating expenses, and the uncertainty of cash flows between the policyholder and the company. This study focuses on the first of these sources of risk and abstracts away from the other two.

A simple chance-constrained model is presented which integrates the basic economic variables relevant to life insurance company portfolio management. In this model, the firm's objective is to maximize its rate of return on its portfolio subject to a probabalistic solvency constraint, legal constraints, a balance sheet constraint, and non-negativity constraints. Given parameters for the model, the optimal portfolio may be specified, and a sensitivity analysis to parameter changes provides the

basis for several testable hypotheses.

Several significant economic relationships were found using an econometric analysis of a cross-sectional/time series panel of 92 large U.S. life insurance companies over the 1957-71 time period. The amount of surplus and a proxy for the level of financial yields were positively related to investments in equities and negatively related to investments in bonds and mortgages.

Company size was positively related to the proportion of assets invested in common stock and bonds and inversely related to the proportional investment in mortgages. Distributed lag models showed the response to changes in independent variables to be fairly rapid, with the demand for bonds and common stock possessing the highest adjustment rates and changes in the amount of surplus causing the most rapid responses.

While New York regulated companies generally invest in less common stock, this result is largely explained by the economic differences between New York licensed companies and other companies. Stock companies invested in more conservative portfolios than mutuals, given the independent variables: this result is expected because of the differences in the life insurance products sold by the two types of firms. Considerable unexplained interfirm variation persisted, perhaps because the high substitutability among financial assets makes prediction difficult.

Relative yields often did not prove to be significant determinants of portfolio choices. These results may be due to the problems of multicollinearity, autocorrelation, and sample size, but the poor performance of yields may have other bases, as well. In the chance constrained model, the demand for a security is a complex function and, conceptually, the demand for a security may be inversely related to its own yield. The demand for a security will be inelastic with respect to yield changes when upper or lower bound constraints are binding. In addition, the normative significance of relative yields may be questionable if yield changes occur in response to other characteristics of securities such as maturity, liquidity, and risk of default when these other features are not explicitly accounted for in the model.

This study of life insurance portfolio behavior deals with a number of important theoretical and empirical problems. While the empirical results generally conform to expectations derived from an analytical model, much portfolio variation remains unexplained. At the same time, however, much portfolio variation among the large life insurance companies in this study does appear to have a rational economic basis.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

CHAPTER I

INTRODUCTION

A fundamental assumption of microeconomic theory is that economic units are engaged in some optimization process. For example, consumers are assumed to maximize their satisfaction while firms maximize their profits or their wealth. These assumptions lead directly to empirically testable hypotheses concerning the behavior of the economic unit. The application of the optimization process to the behavior of life insurance company portfolios poses a number of important empirical and theoretical problems.

In order to provide a perspective for studying a financial intermediary such as life insurance companies, it is worthwhile to contrast the properties of tangible and financial assets.[1] Tangible assets are highly specialized in form and use, are held primarily for the physical services they yield directly, are highly illiquid, are highly complementary, and are characterized by externalities in use. In comparison, financial assets are generalized, income-earning claims on future production, are highly substitutable, and are much more liquid. It follows from these characteristics that

---

[1]Basil J. Moore, <u>An Introduction to the Theory of Finance: Assetholder Behavior Under Uncertainty</u> (New York: The Free Press, 1968), pp. 11-2.

financial markets should possess more allocational and operational efficiency than real markets; financial markets should commit scarce resources to those alternative uses promising the highest rates of return and at low transactions costs more readily than real markets.

Life insurance companies are financial intermediaries that obtain loanable funds from their policyholders and invest them in a broad range of capital market securities. Life insurance companies are complex organizations whose behavior cannot be explained by a simple, unidimensional goal such as pure profit-maximization because the management of a life insurance company must deal with tradeoffs between the yield on its portfolio and its maturity structure, liquidity, and risk. The purpose of this study is to build a model of efficient portfolio choices available to life insurance companies, given both the economic and institutional frameworks in which they operate, and to test empirically hypotheses derived from this model. This requires an integration of the economic significance of the contractual obligations between insurance companies and policyholders into a model of portfolio choice relevant to the life insurance industry.

Chapter II provides the economic and institutional background upon which the conceptual model and the empirical analysis of this study depend. Since investment decisions are made under risky conditions, the basic sources of risk to life insurance companies are analyzed. The net cash flows between the life company and its policyholders are uncertain because of the uncertainty of mortality predictions and because many options about the disposal of a policy are made at the discretion of the policyholder. Investment net cash flows are necessarily risky because of the basic riskiness of the securities purchased. Future operating expenses are uncertain due to inflation and changing technology. The impacts of the tax and regulatory environment affecting life insurance investment choices are embodied in Federal income taxes, the valuation procedures of the National Association of Insurance Commissioners, state qualitative and quantitative restrictions on portfolio composition, and other taxes and regulations.

In Chapter II, a chance-constrained model is developed in which a life company's objective is to maximize its rate of return on assets subject to the constraint that it has a high probability of remaining solvent and also subject to legal, non-negativity, and balance sheet constraints. According to the model, a life company's capacity to undertake risks and probably remain solvent is positively related to the company's amount of surplus and its rate of return on assets and negatively related to the rate the company pays on its liabilities. An increase in a company's risk-taking capacity (through a change in the above), a relaxation of a binding legal constraint on a risky asset, or a decrease in management's degree of risk-aversion should result in a portfolio shift towards relatively riskier types of securities.

Chapter IV empirically tests some of the economic relationships derived from the chance-constrained model of Chapter III. The statistical base is a cross-section/time-series panel of 104 large life insurance companies over 15 years from 1957-1971. The basic results obtained from ordinary least squares models and analysis of covariance techniques generally confirm the hypotheses obtained from the chance-constrained model. These results are extended by using a technique to remove first-order autocorrelation and by applying the Koyck and the Almon distributed lag models to explore the dynamic properties of the portfolio adjustment process.

In Chapter V, a summation of the material discussed in preceding chapters is presented and conclusions drawn on the basis of that information are reviewed. The performance of the life insurance industry is an economic concern not only to the companies and their regulators, but to policyholders and the capital markets in general. In addition to the conclusions specific to the life insurance industry, some conceptual and empirical results have significant implications for financial research in other areas.

CHAPTER II

THE LIFE INSURANCE INVESTMENT PROCESS

In Chapter II, we consider the economic and institutional factors that affect the investment decisions of life insurance companies. Because a company's mortality experience, its operating expenses, and its investment performance are uncertain, the life insurance company is making decisions under risk, with the last of the three sources of risk above chosen as the focus of this study. In addition, the tax and regulatory environment of the industry ordain many of the characteristics of the framework in which life insurance companies optimize.

Rational economic behavior must be consistent with basic objectives. In order to evaluate this behavior, goals must be made explicit, resources, constraints, and economic relationships must be recognized, and economic performance must be measured. Inconsistent behavior is demonstrated by life insurance companies when they exhibit wide variances in holdings of certain capital securities. Such behavior must be explained by disagreement about basic investment goals, by different economic environments, or by ineptitude of portfolio managers.

Differences over basic objectives undoubtedly exist, as suggested by Wehrle who summarizes three views of life insurance investment.[1] The first, which he calls the capital security portfolio, places primary emphasis on safety and minimization of default risk, secondary emphasis on high yields and even less emphasis on liquidity. In a second view, the income security portfolio, long-run solvency through matching the maturity of its assets as nearly as possible to the maturity of its policy liabilities is stressed. This view minimizes income risk due to future interest rate movements, sacrifices yield in order to go as long as possible, and attaches little worth to liquidity balances. The third view is the competitive or yield portfolio; it seeks the highest possible yield consistent with increases in default risk or income risk. Under the competitive portfolio, firms will trade along the yield curve to maximize returns, going long when interest rates are at cyclical highs and going short when yields are at cyclical lows. To accommodate such portfolio shifts, the firm must maintain a certain level of liquidity.

Not only may different firms have differing objectives, but the objectives of a given firm may change through time in response to changes in capital market conditions,

---

[1]Leroy S. Wehrle, "Life Insurance Investment--the Experience of Four Companies," Yale Economic Essays, I (Spring, 1961), 70-136.

in the firm's size and growth rate, in regulatory constraints or tax laws, and in the composition of its accumulated stock of assets. Investment philosophies are affected by competitive pressures from other life insurance companies as well as from other financial intermediaries.

The environment in which LICs operate in the United States appears fairly homogeneous, with some differences in the regulations and taxes of the 50 states. More importantly, because of only slight immobility in mortgage markets, all firms in the U.S. should have access to similar securities, and there must be some rational basis for portfolio differences. In contrast, it would be difficult to make comparisons of operating efficiency across national lines or between widely separated periods of time because of fundamental differences or changes in capital markets. Evaluation of company or industry performance must accurately reflect relevant state regulations, taxation, and other constraints. Changes or differences in any of these aspects of the economic environment should result in different economic behavior.

Allowing for the selection of different objectives and the variation in the current economic environment, mismanagement is the final factor resulting in different portfolios. The implication here is not that all portfolios should be identical, but that management's behavior must be consistent with its own objectives. Management must be capable of balancing the marginal contributions of every investment toward overall performance; this requires understanding the economics of the capital market, the legal framework within which companies operate, and portfolio theory. In addition, it is possible that some companies are used as vehicles to serve private interests in a manner inconsistent with the long-run goals of the LIC or its policyholders. Regardless of the reason, mismanagement should eventually show up in portfolio performance.

The portfolio choices of LICs vary because of differing objectives, constraints, and, perhaps, mismanagement. The model of life insurance portfolio behavior developed in this study is sufficiently general that it should be applicable to United States and Canadian companies, and, at the same time, the model possesses enough empirical content to permit evaluation of performance. This study argues for a model which assumes that life companies maximize their long-run rate of return subject to solvency (survival), legal, and other relevant constraints.

Sources of Risk

Like any other business, LICs operate under conditions of risk specific to their industry; three basic sources of risk are the uncertainty of the net cash flows between the LIC and its policyholders, the riskiness of the securities it purchases, and the uncertainty of future operating costs. The pattern of net cash flows between the LIC and its policyholders is altered by changes in mortality (which change the stream of premium inflows and death benefit outflows), by policy loans, by surrenders, and by other options exercisable by policyholders. Various factors such as default risk, callability, liquidity, uncertainties about taxation, inability to diversify efficiently, and the nondiversifiable risk of the securities contributes to the riskiness of the assets held by LICs. Future operating expenses can change due to inflation, technology, and institutional changes. The nature of the life insurance policy ordains the sources of risk to life insurance companies. The premiums paid on a life insurance policy are figured in the three following steps:

(1) For a given policy, the expected death benefit payment in each future period throughout the policyholder's life is estimated using a standard mortality table such as the Commissioners 1958 Standard Ordinary Mortality Table.

(2) The present value of the stream of expected death benefits is found by discounting the expected cash outflows by an "assumed rate" of return on the life company's investments. The "net premium" is then calculated by finding an expected periodic premium inflow (for all living policyholders) whose present value, discounted

back at the same assumed rate, equals the present value of future death benefits. A high assumed rate results in a lower net premium and a slower accumulation of reserves than would a lower rate.

(3) A loading factor is used to increase the "net premium," arriving at the "gross premium," which is the actual premium paid by policyholders. The load must be sufficient to provide for all operating expenses (such as commissions, wages and salaries, rent, taxes, etc.) as well as profit.

Life insurance company income comes from free investment income and underwriting gain, both of which are subject to risk. Free investment income occurs if the actual yield on investments exceeds that yield assumed for setting premiums and reserves. One source of underwriting gain is the mortality gain, which occurs when the company's actual mortality experience is more favorable than the mortality tables used in setting the premiums, thereby prolonging the payment of premiums and deferring the payment of death benefits. The other potential source of underwriting gain is loading, which may exceed operating expenses. The three sources of risk directly affect free investment income and underwriting income and, consequently, the long-run solvency of the LIC.

Uncertainty of net cash flows between life insurance companies and their policyholders

Deviations from expected patterns of mortality obviously affect the cash flows between the LIC and policyholders. Earlier deaths reduce the premium inflows and accelerate the payment of death benefits and, conversely, later deaths extend the receipt of premiums and delay death benefits. An epidemic could present companies with a mass of death claims at an unexpected time. While wartime deaths may not obligate LICs to pay the face value of the policies, the companies, nevertheless, must return the accumulated surrender values to estates of policyholders who die as a result of war. Mortality patterns change slowly, but they remain a source of risk to the LIC.

Furthermore, LICs are obligated to make policy loans at legally prescribed rates (usually 6%) at the discretion of the policyholder. The LIC must forego certain investments in other assets in order to accommodate the demand for policy loans. The demand for policy loans is, of course, part of the demand for loanable funds by policyholders and is related to the cost and availability of alternative sources of credit. Schott found the demand for policy loans to be a function of the yield on 6 month prime commercial paper, the percentage change in the money supply (narrowly defined), consumer installment credit outstanding, and the change in GNP.[2] The belief that policy loans are related to unemployment is expressed by Brimmer.[3] Schott concludes ". . . that interest rate variations are the most important single influence on policy loans and have the most immediate and largest impact."[4] The amount of policy loans outstanding is highly variable, primarily depending on short-term interest rates. From 1954-1965, increases in policy loans represented 6.12% of the increase in assets and the corresponding figure for 1965-1970 is 17.34%. Since policy loans are likely to be greater at higher interest rates, the ability of insurance companies to invest as much as they wish in long-term securities at secular or cyclical high interest rates is restricted; conversely, they have relatively more funds to invest at low rates (when the demand for policy loan extensions or renewals decreases and repayments increase).

The LIC must also maintain liquidity to meet cash surrenders of policies; moreover, surrenders are difficult to forecast, although their economic function is understood. Cash surrenders should increase in times of

---

[2]Frances H. Schott, "Disintermediation Through Policy Loans at Life Insurance Companies," Journal of Finance, XXVI (June, 1971), 719-29.

[3]Andrew F. Brimmer, Life Insurance Companies in the Capital Market (East Lansing: Michigan State University Press, 1962), p. 37.

[4]Schott, 727-8.

economic crisis, such as high unemployment.[5] In addition to the need for emergency funds, policyholders draw down savings in life policies to make payments on homes and consumer durables, to pay for business ventures, or to reinvest the funds in stocks or other equities.[6] Cash surrenders can be explained by some of the same economic variables which explain policy loans. It must also be considered that policyholders may be more willing to tap this source of accumulated savings after their need for life insurance diminishes because of less family responsibility, more savings of all forms, and the lower income which may accompany increased age. Thus, in addition to short-run economic variables, long-run demographic and economic factors would be expected to affect cash surrenders.

A number of other arrangements can alter the pattern of premium payments and benefits beyond those mentioned above. The right to buy additional insurance at specific future dates is included in some policies and a disability rider, which automatically pays premiums in case of disability, is a frequent option. Policyholders may stop paying premiums and convert to paid-up insurance instead of surrendering for cash. It is possible for beneficiaries to elect an annuity for life or an annuity based on interest income instead of taking a lump sum. Dividends earned on

---

[5]Brimmer. Ibid.

[6]Ibid., 20.

participating policies (which were $3.58 billion in 1970) may be applied to current premiums, may be left with the company to earn interest, may be used to buy additional insurance, or may be taken out in cash.

Recent experience would suggest that payments other than death benefits are important in magnitude and substantial sources of uncertainty. In 1970, total death payments were $7.02 billion, of which $3.55 billion (or 50.52%) was paid on ordinary policies, while the remainder was paid on group or industrial policies. Not all death benefits were paid in a lump sum; this is illustrated by the fact that $0.79 (11.25% of all death benefits) was left with the company under supplementary contracts. In 1970, the net increase in policy loans was $2.24 billion, which was 61.3% of death benefits on permanent policies. During the same year, surrender values were $2.89 billion, which was 81.4% of death benefits on permanent policies. Also 5.9% of all policies (3.9% of all policies in force 2 years or more) were surrendered. A portion of surrenders was left with the company for paid-up insurance or extended coverage, but about $2.5 billion was paid out in cash.

In attempting to match the maturity of its assets and liabilities, the LIC must know the timing and size of all future net cash flows with its policyholders. However, due to the uncertainty of mortality assumptions, policy loan demands, cash surrenders, and other arrangements, accurately forecasting these net cash flows is an extremely difficult and costly undertaking.

Riskiness of investments

Future net cash flows from investments are risky because the income from, and market values of, securities depend on the long-run solvency of the firms issuing the securities. The bankruptcy of a corporation terminates the periodic payment of interest on its bonds and usually forces the LIC to sell the defaulted bond at a substantial loss. The LIC will then generally purchase new investments. To compensate for the perceived risk of default, investors require a risk premium in the yields on bonds which is a function of the perceived risk. Fisher finds that 75% of the variation in risk premiums among bonds is explained by the earnings variability of the company, by the length of time the company has been solvent and creditors have not taken a loss, by the equity/debt ratio, and by the market value of all publicly traded bonds of the company.[7] The first three variables are related to subjective default risk estimates and the fourth is related to marketability. As expected, Hickman found default rates to be inversely related to agency bond ratings (Moody's and

---

[7]Lawrence Fisher, "Determinants of Risk Premiums on Corporate Bonds," Journal of Political Economy, LXVII (June, 1959), 217-37.

Standard and Poor's),[8] yet Fraine has found the relative yield at issuance to be as good a predictor of loss rates as agency ratings.[9] The apparent clustering of defaults on bonds is a phenomenon of considerable importance; default losses on bonds of different companies are positively correlated and linked to general, as well as industry, economic performance. Bond markets have operated to assign risk premiums to bonds to compensate bondholders for the additional risk of default on lower quality issues.

Another source of risk on bonds is the call feature, which allows the issuer to buy back the bond at a specified price prior to maturity. Practically all corporate bonds are callable, as are some Treasury bonds and some municipals. The call feature operates to the detriment of bondholders if interest rates fall since the issuer can call in a bond issue, paying a call premium, and replace the bonds with new ones at lower interest rates.

> Life company earnings rates suffered substantially from unscheduled portfolio turnover during the period 1935-45. . . . [In addition] the decline in the life insurance industry's share of public utility financing (and the proportion of life

---

[8] W. Braddock Hickman, Corporate Bond Quality and Investor Experience (New York: National Bureau of Economic Research, 1958), p. 176.

[9] Harold G. Fraine, Valuation of Securities Holdings of Life Insurance Companies (Homewood, Illinois: Richard D. Irwin, Inc., 1962), pp. 52-3.

> company funds going into public utility sectors since 1950 was attributed principally to the lack of satisfactory call protection on many utility company debentures.[10]

A straightforward comparison of yields on callable bonds and noncallable bonds is impossible, but Jen and Wert found that bonds with deferred callability had lower yields than those immediately callable, and that this differential was a function of the level of coupon rates (those bonds with high coupons have a higher probability of being called).[11] Thus, bondholders seem to extract a higher yield from issuers in compensation for the risk of the call feature. Preferred stock is frequently callable and mortgages may be repaid at any time, often with a prepayment penalty similar to the call premium. Thus, the bulk of life insurance assets is subject to a form of call and declining interest rates could trigger calls of the above securities, forcing the LICs to reinvest the funds at lower yields.

Another source of risk to LICs is the danger of not being sufficiently liquid. Cash, short-term securities, maturing securities, bond sinking fund payments, mortgage amortizations, and the excess of premium income

---

[10] Lawrence D. Jones, Investment Policies of Life Insurance Companies (Boston: Division of Research, Graduate School of Business Administration, Harvard University, 1968), p. 55.

[11] Frank C. Jen and James E. Wert, "The Value of the Deferred Call Privilege," National Banking Review, III (March, 1966), 369-78.

over expenses and disbursements are sources of cash available, not including liquidation of holdings of long-term investments which may involve a substantial loss. Most evidence suggests that a liquidity crisis is highly unlikely. O'Leary states

> A third characteristic of life insurance companies as investors which I think is pertinent here is the high degree of stability and magnitude of their net cash inflows, which makes the idea of liquidity preference have a little meaning for them. Our studies of the life insurance business indicate that even in the bottom of the Great Depression the life insurance business as a whole generated enough income to meet disbursements without the need to liquidate holdings.[12]

The most severe mortality crisis faced by LICs in this century--the 1918-19 influenze epidemic--apparently was insufficient to force liquidation of assets. Premium inflows were adequate to cover benefit payments over the period.[13] Nine of the largest 14 companies used short-term bank borrowing but the loans were quickly repaid with no liquidation of assets.[14] Walter concludes, "The period of the thirties provided a useful test of the survival power of life companies. For 45 large companies, total cash inflow from premiums and investment income exceeded

---

[12] James J. O'Leary, "The Institutional Saving-Investment Process and Current Economic Theory," American Economic Review, XLIV (May, 1954), 463.

[13] Jack G. Klein, "The Liquidity Structure of Life Insurance Companies" (unpublished Ph.D. dissertation, University of Pennsylvania, 1965), Chap. IV and Appendix B.

[14] Ibid., 82.

total outflow to beneficiaries and living policyholders by 69% in the period 1930 through 1936."[15] It is obvious that the liquidity requirements for life companies are far less than for deposit-type financial institutions and that the only extraordinary need for liquidity is to accommodate shifts in the composition of their portfolios. Liquidity constraints on life insurance portfolios appear trivial compared to those on commercial banks and some other savings institutions.

Another source of uncertainty for LICs is potential changes in their tax status, which could alter their net cash flows. For example, the Life Insurance Company Income Tax Act of 1959 raised the portion of earnings from invested assets subject to Federal Income taxes from 15% to roughly 21-22% and taxed underwriting income for the first time since 1921.[16] The trend and effects of changes can be seen in Table 2-1.

---

[15]James E. Walter, The Investment Process As Characterized by Leading Life Insurance Companies (Boston: Harvard University Press, 1962), p. 8.

[16]Brimmer, Life Insurance Companies, p. 22.

TABLE 2-1

TAXES PAID BY LIFE INSURANCE COMPANIES, 1950-1970

Year	(1) Fed. Inc. Tax	(2) Total Tax	(3) Investmt. Income	(1) ÷ (2)	(1) ÷ (3)	(2) ÷ (3)
1950	20*	205*	2,075*	9.76%	0.96%	9.88%
1955	189	472	2,801	40.04	6.75	16.85
1960	479	920	4,304	52.07	11.13	21.38
1965	741	1,375	6,778	53.89	10.93	20.29
1970	1,232	2,204	10,144	55.90	12.15	21.72
1950-58	1,606	4,024	23,793	39.91	6.75	16.91
1959-70	9,812	17,484	80,163	56.12	12.24	21.81

*millions of dollars

Source: 1961, 1970 and 1971 Life Insurance Fact Books (New York: Institute of Life Insurance), 1961, 1970, and 1971.

The 1959 tax act raised the amount of federal income taxes relative to total taxes and to investment income and also raised total taxes relative to investment income. Changes in future tax structures subject the LIC to a degree of risk since taxes represent a net cash outflow beyond their control.

Inefficient diversification may be considered another source of risk for LICs. Suppliers and demanders of loanable funds simultaneously price individual financial assets in the capital markets.[17] If for legal or traditional

[17]For a simple explanation of price determination of risky capital assets, see William F. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," Journal of Finance, XIX (September, 1964), 425-42.

reasons, LICs do not choose the assets which best allow them to meet their objectives, they are unnecessarily subjecting themselves to additional risks. Compared to other financial intermediaries, life companies are characterized by long-term investment objectives, stable cash inflows, and few liquidity problems; therefore, they may have a comparative advantage in some financial markets, especially in long-term markets. While the economic means may be available to achieve or approach more closely their economic objectives, LICs may fail to utilize these means due to mismanagement or ill-conceived regulation. Some economists[18] argue that capital markets can be in substantial disequilibrium. Eckstein finds ". . . the capital market to be imperfect, to be rife with rationing, ignorance, differential tax treatment, reluctance to finance investment from external funds, slow adjustment processes, etc., which destroy the normative significance of actual rates found in the markets."[19] LICs should be

---

[18] Jack Hirschlaefer, "Efficient Allocation of Capital in an Uncertain World," American Economic Review, LIV (May, 1964), 77; Otto Eckstein, "A Survey in the Theory of Public Expenditure Criteria, Reply," in Conference of the Universities--National Bureau Committee for Economic Research, Public Finance: Needs, Sources, and Utilization (Princeton: Princeton University Press, 1961), 493-4; and Marc Nerlove, "Factors Affecting Differences Among Rates of Return on Investments in Individual Common Stocks," Review of Economics and Statistics, L (August, 1968), 312-31.

[19] Eckstein, "Public Expenditure Criteria," 493.

able to move quickly to exploit any market imperfections which are to their benefit.

Besides some of the previously mentioned sources of risk, the attainment of long-run objectives depends on the yield obtainable on future net cash flows, whatever their source. For example, an investment in a bond which yields 7% to maturity actually has a compound growth rate of 7% to maturity only if interest payments received up to maturity are reinvested at a rate of return of 7%. Likewise, if the investment time horizon exceeds the maturity of a fixed-income security, the yield at which the principal must be reinvested is risky. The yield on preferred stock is subject to risk due to variations in the market price of the shares and the yield on common shares varies due to changes in both future dividends and market prices.

Uncertainty of operating expenses

Since 1950, operating expenses (commissions, agency costs, home office costs, medical fees, and rents) have averaged between 16% and 18% of income (premiums, net investment income, and other income).[20] These expenses can be split into those associated with selling and issuing new policies and those associated with maintaining

---

[20] Institute of Life Insurance, 1971 Life Insurance Fact Book (New York: Institute of Life Insurance, 1971), p. 64.

old policies. The commissions, examinations, and other expenses on new policies are recovered slowly out of the excess of gross premiums over net premiums. In order to offset partially the dangers of mass early surrenders, LICs do not credit policyholders with their full surrender cash reserves until the contract is 10 to 15 years old. Future operating expenses involved in maintaining already existing policies are a source of risk because many LIC cash inflows and outflows are fixed in nominal dollars, while operating expenses are not. One writer has suggested that hedging the general inflationary pressure on future operating expenses would require, by itself, equity investments totaling 7 to 8 percent of assets.[21] Changes in technology, such as increased use of electronic data processing equipment, may offset some of the increases in operating expenses. Most companies have shifted away from policies which have high administrative costs (such as industrial life insurance) in order to reduce their risk exposure.

Portfolio decisions are primarily affected by the riskiness of investments, but, since the objective of portfolio management is to help the life company fulfill its obligations to policyholders, the other sources of risk and the tax and regulatory environment must constrain and guide investment choices.

---

[21]W. M. Anderson, "The Long View of Life Insurance Investment," American Life Convention General Proceedings (Chicago: American Life Convention, 1954), p. 367.

Tax and Regulatory Environment

In addition to the sources of risk dealt with earlier in this study, portfolio choices are subject to exogenous forces, including federal income taxes, the National Association of Insurance Commissioners valuation procedures, state regulations on the qualitative and quantitative makeup of portfolios, and other taxes and regulations. Each of these affects investment decisions and, consequently, deserves analysis.

Federal income taxation

The Life Insurance Company Income Tax Act of 1959 established two possible tax bases for LICs, namely, "Taxable Investment Income" and "Gain from Operations." If, on the one hand, the Gain from Operations is less than Taxable Investment Income, the tax base is the smaller figure. On the other hand, if Gain from Operations exceeds Taxable Investment Income, the current tax base is Taxable Investment Income plus one-half of the excess of Gain from Operations over Taxable Investment Income. The tax on the other half of this excess is deferred until it is distributed to stockholders. In addition, LICs pay a tax on capital gains.

Taxable Investment Income, termed the "phase I" tax base, consists of gross investment income less investment expenses, policy and contract interest requirements, the company's share of tax-exempt income, and a small business deduction.

The following items define the Gain from Operations, or the "phase II" tax base; gross income (premiums plus investment income) minus deductions for death benefits, increases in policy reserves, cash surrender payments, insurance expenses, the small business deduction, policyholder dividends, the company's share of tax-exempt income, special deductions (2 percent of group premiums and the greater of 3 percent of premiums on nonparticipating contracts or 10 percent of the increase in reserves on those contracts), and loss carryback-carryforward provisions. Since the policyholder dividend deduction is limited to the excess of Gain from Operations over Taxable Investment Income plus $250,000, the tax base for large mutuals (and some stock companies selling participating contracts) will usually be the phase I tax base minus $250,000.[22] The difference between the phase I and phase II tax bases is, generally speaking, the underwriting gain which is not returned to the policyholders as dividends. The 1959 Act closed a loophole wherein underwriting gains were not taxes from 1921 to 1957. The phase II tax computation raised the tax burden on

[22] Stephen M. Kaufman, "The Life Insurance Company Income Tax Act of 1959: An Appraisal of the Effects upon the Life Insurance Industry, Part II," National Tax Journal, XVII (March, 1964), 43; and Bernie E. Abelle, Jr., "An Evaluation of the Life Insurance Company Income Tax Act of 1959," Journal of Insurance, XXX (September, 1963), 419.

companies selling nonparticipating and nonreserved lines of insurance, which are mostly stock companies. The tax on investment income was expected to generate about 90 percent of the taxes on LICs with the other 10 percent coming from taxes on underwriting gain.[23]

That half of the excess of Gain from Operations over Taxable Investment Income on which taxes are paid currently is assigned to the Shareholders Surplus Account (SSA), while the untaxed half is assigned to the Policyholders Surplus Account (PSA). If funds are removed from the PSA to the SSA to pay dividends to stockholders or for any other reason, a phase III tax is paid on the amount of funds transferred out of the PSA. Congress limited the PSA to 15 percent of a company's life insurance reserves or to 50 percent of premium income. The deferral of taxes on a portion of income retained behind other liabilities was a concession designed to encourage the existence of a surplus margin behind insurance contracts. Normally, the phase III tax collections will be dwarfed by the magnitude of the phase I and phase II collections.

In addition, long-term capital gains are taxed separately at the capital gains rate, unless there are operating losses. In this case, capital gains may be used as an offset to such losses. Gains on which taxes have been paid are assigned to the SSA.

---

[23]Andrew F. Whitman and Howard E. Thompson, "The Impact of the 1959 Income Tax Act on Stock and Mutual Companies: A Simulation Study," Journal of Risk and Insurance, XXXIV (June, 1967), 217.

Since the impact of the 1959 tax act can be seen largely through the phase I tax computations, investment decisions are affected primarily by this portion of the act. In order to help establish the nature of the phase I tax computation, the following symbols are used:

T = phase I income tax

I = investment yield

P = policy interest requirement

R = mean life insurance reserves

R_A = adjusted life insurance reserves

A = mean assets

Q = reduction items

D = dividends received

E = tax-exempt interest

r = average assumed rate of return on R

i = I/A

m = marginal corporate tax rate

All symbols denoted by capital letters are in dollars.

A LIC's taxable investment income is simply its investment yield less its policy interest requirement and less certain reduction items. The tax liability is

$$T = m(I - P - Q) - 25{,}000(m - .22). \tag{1}$$

Taxes are reduced by $25{,}000(m - .22)$, since the first \$25,000 of corporate income is taxed at the lower marginal rate of 22 percent.

The policy interest rate requirement is found by multiplying the actual rate of return on assets times adjusted reserves.

$$P = i \cdot R_A \tag{2}$$

In the 1959 law, adjusted reserves are defined as

$$R_A = (1 + 10r - 10i)R. \tag{3}$$

By substitution,

$$P = i(1 + 10r - 10i)R = (i + 10ir - 10i^2)R. \tag{4}$$

A company with R = \$1,000,000, r = .03, and i = .04 would have adjusted reserves of .90R = \$900,000 and a policy interest requirement of \$36,000, even though it actually owes its policyholders 3 percent of \$1,000,000 or \$30,000.

The reduction items are

$$Q = \left(\frac{I - P}{I}\right)(E + .85D) + 25{,}000 \tag{5}$$

$\frac{I - P}{I}$ is the company's share of investment income; E is interest received on municipals; and .85D is the amount of the 85 percent intercorporate dividend exclusion. If the company's share of investment income is 30 percent, only 30 percent of tax-exempt interest and excluded dividends may be used to reduce taxable income. This is one unique feature of the tax which obviously discriminates against LICs since any other corporate investor can deduct 100 percent of such income from its taxable income. Life companies also have a small business deduction of the lesser of 10 percent of investment yield or \$25,000. Equation (5) assumes investment income of at least \$250,000.

Substituting equations (4) and (5) into equation (1) gives us the tax on investment income.

$$T \doteq [I - (i + 10ir - 10i^2)R - (\frac{I - P}{I})(E + .85D) - 25{,}000] \cdot m - 25{,}000(m - .22) \tag{6}$$

Clearly,

$$T = \{I - [\frac{I}{A} + 10\, r\, (\frac{I}{A}) - 10\, (\frac{I}{A})^2]\, R - [\, 1 - (1 + 10r - 10\,\frac{I}{A}) \cdot \frac{R}{A}\,]\, (E + .85D) - 25{,}000\} \cdot m - 25{,}000(m - .22) \tag{7}$$

The marginal tax rate on fully taxable income is found by taking the partial derivative of (7) with respect to I.

$$\frac{\delta T}{\delta I} = [1 - (1 + 10r - 20\frac{I}{A})\,\frac{R}{A} - \frac{10R(E + .85D)}{A^2}] \cdot m \tag{8}$$

$$\frac{\delta^2 T}{\delta I^2} = 20\,\frac{R}{A^2} \cdot m \tag{9}$$

Equation (7) is concave from above and reaches a minimum where $\frac{\delta T}{\delta I} = 0$. At levels of I greater than the I where T is minimized, $\frac{\delta T}{\delta I}$ will be monotonically increasing. The marginal tax rate depends on the parameters in equation (7) which apply to a particular company. $\frac{R}{A}$ is less than unity for a solvent company, averaging .93 for mutuals and .86 for stock companies. (1 + 10r - 20i) is also less than unity as long as i >r/2; this has never failed to happen. 10(E + .85D)/A has been very small historically. Thus, the marginal tax rate of LICs is positive and progressive for a wide range of parameter values about those which currently exist. (This progressivity can also be established by finding the elasticity of taxes with respect to income.)

The manner in which the tax-deductible policy interest requirement is determined is responsible for the tax progressivity. The only non-linear term in equation (7) is in the policy interest requirement. When income changes, the policy interest requirement changes as well, so $\frac{\delta T}{\delta I}$ will depend on the elasticity of P with respect to I.

$$\frac{\delta P}{\delta I} = (1 + 10r - 20i)\ \frac{R}{A} \tag{10}$$

$$\frac{\delta P}{\delta I} \cdot \frac{I}{P} = \frac{(1 + 10r - 20i)}{(1 + 10r - 10i)} \tag{11}$$

$\frac{\delta P}{\delta I} \cdot \frac{I}{P} < 1$ for any positive i. In addition, $\frac{\delta P}{\delta I} \cdot \frac{I}{P} > 0$ as long as $1 + 10r > 20i$. If $0 < \frac{\delta P}{\delta I} \cdot \frac{I}{P} < 1$, an increase in income increases a tax deduction by a fraction of that increase and the marginal tax rate will be less than the corporate rate. Since the proportional increase in P is less than the proportional increase in income, the fraction of income subject to a constant rate increases, and the tax is, therefore, progressive. If E and D are zero, the marginal tax rate will equal the corporate rate where $20i = 1 + 10r$. If $r = .03$, this occurs when $i = .065$. At this time $\frac{\delta P}{\delta I} \cdot \frac{I}{P} = 0$. If i increases further, the marginal tax rate continues to increase.

The marginal tax on tax-exempt interest is found by taking the partial derivative of (8) with respect to E. Since E is included in I, $\frac{\delta I}{\delta E} = 1$.

$$\frac{\delta T}{\delta E} = \{1 - (1 + 10r - 20i)\frac{R}{A} - \frac{10R(E + .85D)}{A^2} - [1 -(1 + 10r - 10i)\frac{R}{A}]\}\cdot m \tag{12}$$

$$\frac{\delta T}{\delta E} = \frac{\delta T}{\delta I} - (\frac{I - P}{I}) \cdot m \tag{13}$$

According to equation (13), the marginal tax rate on tax-exempt interest is the marginal tax rate on fully taxable income less the product of the company's share of investment income and the marginal corporate tax rate. If $\frac{I - P}{I} = .25$ and $m = .48$, $\frac{\delta T}{\delta E}$ is .12 less than $\frac{\delta T}{\delta I}$. The greater the company's share of investment income, the greater the differential absolute advantage of tax-exempt income. However, only if the company's share is 100 percent does the LIC enjoy the full tax exemption. Similarly, the marginal tax rate on dividends is

$$\frac{\delta T}{\delta D} = \{1 - (1 + 10r - 20i)\frac{R}{A} - \frac{10r(E + .85D)}{A^2} - .85[1 - (1 + 10r - 10i)]\frac{R}{A}\}\cdot m \tag{14}$$

$$\frac{\delta T}{\delta D} = \frac{\delta T}{\delta I} - .85\,(\frac{I - P}{I}) \cdot m \tag{15}$$

The same comments which apply to tax-exempt interest apply to dividends, except the absolute advantage of dividend income over taxable income is 85 percent as large as that of tax-exempt interest over taxable income.

A numerical example illustrating the marginal tax rates on I, E, and D appears in Table 2-2. Company B is identical to Company A except that it has $30,000 of additional fully-taxable income; Company C has $30,000 of additional tax-exempt interest, and Company D has an additional $30,000 of dividends. The marginal tax rates for Company A on I, E, and D, computed using equations (8), (12), and (14), were .3075, .1675, and .1885, respectively, which are approximately the same as the incremental tax rates computed in Table 2-2. The relationships in the mathematical explanation of the life company tax are illustrated by this numerical example.

If the current earnings rate exceeds the average of the previous four years, the policy interest requirement is computed somewhat differently. The change can be seen by substituting

$$i' = .8\bar{i} + .2i \tag{16}$$

for i in equations (2), (3), and (4), and then consistently using these new definitions of P and R_A wherever they subsequently appear. $\bar{i}$ is a simple average of the current earnings rates for the previous four years; i is the average for the current year, while i' is the five-year moving average including the current year. This complicates the analysis slightly by introducing an additional parameter ($\bar{i}$) and it slows down the progressitity of the tax rate.

TABLE 2-2

EXAMPLE OF TAXATION OF LIFE COMPANY INCOME

	Company A	Company B additional fully taxable interest income	Company C additional tax exempt interest income	Company D additional dividend income
(1)	1,350,000	1,380,000	1,380,000	1,380,000
(2)	60,000	60,000	90,000	60,000
(3)	40,000	40,000	40,000	70,000
(4)	30,000,000	30,000,000	30,000,000	30,000,000
(5)	.045	.046	.046	.046
(6)	.03	.03	.03	.03
(7)	25,000,000	25,000,000	25,000,000	25,000,000
(8)	21,250,000	21,000,000	21,000,000	21,000,000
(9)	956,250	966,000	966,000	966,000
(10)	.2917	.3	.3	.3
(11)	52,417	53,200	62,200	60,850
(12)	341,333	360,800	351,800	353,150
(13)	157,340	166,684	162,324	163,012
(14)	-	9,344	4,984	5,672
(15)	-	.3115	.1661	.1891

(1) = investment income = I
(2) = tax-exempt income (included in (1)) = E
(3) = dividends (included in (1)) = D
(4) = mean assets = A
(5) = (1)/(4) = current earnings rate = i
(6) = average reserve interest rate = r
(7) = policy reserves = R
(8) = adjusted policy reserves = R_A
(9) = policy interest requirement = P

(10) = company's share of investment income = $\frac{I - P}{I}$

(11) = reduction items = (10).[(2) + .85(3)] + 25,000 = Q
(12) = taxable investment income = (1) - (9) - (11)
(13) = tax = .48[(12) - 25,000] + 22(25,000) = T
(14) = incremental tax on 30,000 increase in income
(15) = incremental tax rate = (14) ÷ 30,000

When the marginal tax rate is viewed as a function of i, a kink appears at $i = \bar{i}$, as in Figure 2-1.

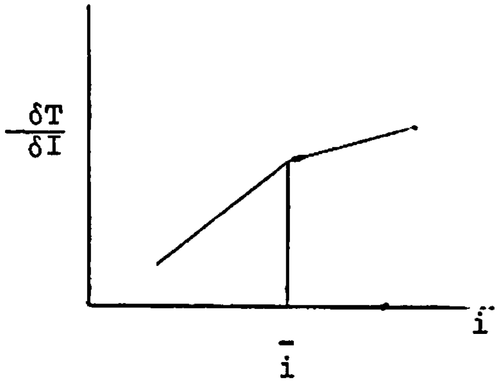


Fig. 2-1.--Marginal tax rate as a function of the current rate of return on assets

Since i' is a moving average and since i is very stable (resulting from years of accumulated investment decisions), it is unlikely that $\frac{\delta T}{\delta I}$ will reach the full corporate rate in the near future, given current parameters.

The impact of federal taxation on investment decisions is complex; an examination of some of the factors involved seems to be relevant here. In the first place, progressive taxation reinforces conservative investment behavior and transforms the before-tax risk-return choices in such a way that a risk-averse investor will choose a lower return, lower risk set of assets than he would have chosen were taxes not progressive.

Secondly, the effective partial exemption of interest on municipals can cause their after-tax yields to be generally inferior to after-tax yields on corporates of similar quality. If the marginal tax on investment income averages .30 and the company's share of investment income averages .25, then the marginal tax on tax-exempt interest is .18, and the before-tax yield on municipals must be at least 83.3 percent of the before-tax yield on corporates to have a superior after-tax yield. Since the yield on municipals seldom approaches this fraction, LICs are generally not interested in them.

A third point to be considered is that the federal income tax treatment of preferred stock held by LICs places them in a less favorable position than other corporations because LICs do not enjoy the full benefit of the intercorporate dividend exclusion and, consequently, have lower after-tax yields on preferred stock than other corporate investors. Inferior risk, liquidity, valuation, and quantitative restrictions, in conjunction with the proration of dividend income between company and policyholders, have kept life company holdings of preferred stock low historically.

A fourth consideration deserving mention is that common stock is given favorable treatment over bonds. The return on common stock can be divided into a dividend (current) yield and a yield through share-price appreciation. The dividend component of the yield is taxed at lower marginal rates than fully taxable interest; the tax on capital gains will usually be lower than the marginal rate on interest and can be deferred until the capital gains are actually realized. However, since life insurance companies enjoy only a fraction of the intercorporate dividend exclusion, LIC's relative preference for common stock is not as strong as it would be otherwise.

A fifth and final item that attention should be drawn to is that the treatment of the real estate investment is generally unfavorable. Much LIC real estate investment has been sale-leaseback or purchase-leaseback arrangements which closely resemble mortgages. The attractiveness of leasing is necessarily tied to Internal Revenue Service guidelines regarding the expense allocation depreciation. If the depreciation deduction is reduced by increasing the depreciable life of an asset, the portion of the lease payment which is considered a non-taxed return of principal decreases. Therefore, leases may be less flexible and more heavily taxed than mortgages.[24] However, this conclusion is not a result of the 1959 tax act alone, but of numerous other IRS regulations.

As demonstrated by the preceding discussion, it can be concluded that federal income taxation, as codified in the Life Insurance Income Tax Act of 1959, drives deeply

---

[24] See Kaufman, "Life Insurance Company Income Tax Act, Part II," 53-5.

into general investment philosophy and also has a strong impact on concrete choices among investment alternatives.

NAIC valuation

The National Association of Insurance Commissioners sets valuation standards for LICs in the United States designed to meet the needs of the industry and to promote uniformity across companies. In order to understand the impact of the valuation standards on portfolio choices, a brief description of the treatment of bonds, mortgages, common stock, preferred stock, real estate, and policy loans is pertinent to the study at this point.

All bonds in the top four rating categories of one of the three major rating agencies or bonds meeting one of two more liberal earnings tests qualify for valuation at amortized cost. If a bond is ineligible for amortization under the above or is in default with respect to interest or principal, it must be carried at market value. This treatment is justified by assuming that bonds are held to maturity. Furthermore, alternative valuation systems for non-marketable private placements could prove unsatisfactory. By this device, bond values and bond rates of return are extremely stable on LIC financial statements. As with bonds, mortgage values and yields are also extremely stable under their valuation scheme. Mortgage loans are valued at the original principal less all repayments of principal and no quality tests must be passed to qualify. Common stock is carried on the books at year-end market values, but yields are based on dividend yields alone. Preferred stock has been placed on the books at original cost since 1965. From 1957-1965, preferred stock was valued according to a weighted moving average of year-end market values called the "one-fifth rule." Yields are dividend yields alone and, along with statement values, are very stable. Real estate is valued at cost plus capital improvement less accumulated depreciation. When it is acquired through foreclosure of mortgages, real estate is carried at the amount of the debt. If market values are less than book value, the real estate should be written down. Book values of real estate are stabilized, as are yields, and both are subject to depreciation schedules and other allocations. Policy loans are carried at face value, and, finally a number of accrual and deferred asset and liability accounts peculiar to the life insurance industry also exist.

Perhaps the most unusual account on the right side of the balance sheet is the mandatory securities valuation reserve, which is merely an allocation of surplus intended to absorb default losses or capital gains and losses in order to minimize their impacts on surplus. The annual allocations into the reserve are .05 percent of almost all amortizable bonds up to a cumulative total of 1 percent of such bonds and 1 percent of amortizable bonds failing an earnings test ("Test 2"), bonds in default, preferred stock, and common stock up to a total of 20 percent of such assets, plus all realized or unrealized net capital gains. Contributions to the reserve against common stock cease at 20 percent, but capital gains must be credited to the reserve until 30 percent is reached.

Since the value of a high quality bond and preferred stock portfolio varies inversely with the value of a common stock portfolio over the business cycle,[25] the use of different valuation methods for the securities will result in systematically biased balance sheets over the business cycle. This bias is a result of the inconsistent valuation procedures applied to different categories of investment.

The rate of return figures supplied by LICs on assets and on investment classes are necessarily tied to the idiosyncrasies of the valuation process. These rates of return are subject to the historical accident of when net cash inflows had to be invested, and this is only partially influenced by investing staffs. Walter adds,

---

[25]Alden C. Olson, The Impact of Valuation Requirements on the Preferred Stock Investment Policies of Life Insurance Companies; Occasional Paper No. 13 (East Lansing, Michigan: Bureau of Business and Economic Research, Graduate School of Business Administration, Michigan State University, 1964), p. 101.

> As an exclusive measure of comparative performance, the composite rate of return on invested assets is deficient. It weights the results of past investment decisions more than those attributable to current behavior and is affected by asset growth over which the investment staff has but limited control. It further omits capital gains and losses and reflects arbitrary expense allocations.[26]

Many of the valuation standards apparently were selected for convenience. Bonds are subject to quality scrutiny and contributions to the mandatory security valuation reserve are made on bonds. Mortgages, which as a group are riskier than bonds, are not written down to market value until default actually occurs, and there is a tendency to carry the foreclosed property at the outstanding balance on the debt. No contributions to the mandatory securities valuation reserve are made on mortgages. Preferred stock has a reservation rate of 1 percent compared to .05 percent for bonds, even though the default experience on preferred stock held by LICs has not been inferior to that on bonds.[27] No contributions to the reserve or quality standards exist for real estate.

Fraine presents a list of sixteen inconsistencies of the valuation procedure,[28] which extend far beyond those mentioned above. An understanding of these inconsistencies prompted Jones to write,

---

[26] Walter, The Investment Process, p. 12.

[27] Fraine, Valuation of Securities, p. 95.

[28] Ibid., pp. 163-5.

> One of the consequences of a badly designed valuation system is that life companies will have to formulate policy with the "system risks," as well as "real risks" in mind. As we have seen, there is abundant evidence of serious weakness in the asset valuation regulations concerning life insurance companies. Therefore this part of the external environment can be expected to have had an impact upon portfolio selections.[29]

Espie further criticizes the impact of NAIC valuation requirements by saying,

> The fact that these conventions may be unrealistic in individual company situations is subordinated to their usefulness and simplicity in demonstrating solvency to regulatory authorities. Further subordinated is the fact that when such conventions overvalue liabilities and undervalue assets by unknown amounts--as compared with theoretical but unknown exact valuation accuracy--surplus is distorted, and earnings, which reflect the change of surplus from one year-end to another, may be doubly distorted.[30]

The accounting procedures promulgated by the National Association of Insurance Commissioners undoubtedly are extremely useful for some purposes, but they may obscure the real economic behavior of the firm and, in some cases, interfere with investment decision-making.

State qualitative and quantitative portfolio restrictions

The investments of life companies are regulated by the states in which they are chartered; a few states dominate the regulation of life assets either directly or

---

[29] Jones, Investment Policies, p. 143.

[30] Robert G. Espie, "Financial Statements," in Life Insurance, rev. ed. by Dan M. McGill (Homewood, Illinois: Richard D. Irwin, Inc., 1967), p. 861.

indirectly. For example, life companies chartered in New York own about 30 percent of the industry's assets; those not chartered in New York that are licensed to sell insurance there are required to "substantially comply" with New York insurance laws. Companies licensed to sell insurance in New York hold over two-thirds of industry assets. In addition, insurance commissions in other states may pattern their regulations after New York's laws. While control of life companies might seem quite decentralized and subject to fifty different sets of regulations, a few states actually regulate the vast majority of industry assets. Besides New York, the most important states are Connecticut, Massachusetts, New Jersey, Pennsylvania, and Wisconsin.

Several states regulate the quality of securities acquired and their relative quantities, sometimes broken down by security type, by industry, and by individual issuer. The severity of the regulations varies considerably across security types and the emphasis also varies from state to state, with New York generally possessing the most stringent investment statutes. Since these statutes have a direct impact on portfolio choices, a survey of current regulations is appropriate. The regulations are reviewed in the following order: U.S. government securities, state and local securities, corporate bonds, mortgage loans, preferred stock, common stock, real estate, and leeway clauses.[31]

Investments in obligations of the federal government or in debt guaranteed by the federal government are generally unrestricted, whereas obligations of foreign governments are restricted. For example, the New York laws restrict life holdings to the greater of 150 percent of a company's liabilities in a given country or the amount that country requires the company to invest there. New York companies may invest up to one percent of assets in foreign securities in addition to the above.

Limitations on the quantity of state and local securities owned generally do not exist, although the quality of these securities eligible for investment is regulated. Special assessment bonds, as well as issues in default, are usually forbidden. Often issues of small municipalities are ineligible and the amount of tax-supported obligations cannot exceed the limit of 10 percent placed on all Canadian securities (which is an exception to the laws on foreign securities).

Quantitative restrictions on the proportion of assets devoted to corporate bonds have no operational significance; however, qualitative restrictions vary considerably from state to state. Connecticut and New Jersey demand only that the bonds not be in default while in New York complex earnings requirements depend on the

---

[31]The following is a summary of regulations given in Brimmer, Jones, and Walter, cited earlier.

type of bond. Mortgage bonds have less stringent earnings tests than debentures and contingent interest obligations face even stricter earnings requirements. Generally speaking, unsecured loans to unincorporated businesses are forbidden. It is a fact that some states have easier requirements for railroad and utility bonds than for bonds of other industries.

For the most part, the quantity of bonds of a single issuer is generally limited. In New York, the proportion of assets issued by a single institution cannot exceed five percent and the proportion in a single issue cannot exceed one-half percent. In New Jersey, this limit is 10 percent of surplus, while in Wisconsin no single secured issue can exceed two percent of assets and no single unsecured issue can exceed one percent of assets. The most significant impact of regulations on corporate bonds is the ineligibility of low-quality debt issues.

Mortgage loans guaranteed by the VA or insured by the FHA are unrestricted although conventional mortgages are limited to 40 percent of assets in New York. No single parcel can exceed the greater of $30,000 or 2 percent of assets and mortgages of a single mortgagor cannot exceed 10 percent. On single-family homes, the loan/value ratio cannot exceed 75 percent and loans must be fully amortized in 30 years. On leaseholds, the loan/value ratio must be less than 66-2/3 percent and mortgages must be fully amortized between 21 and 30 years.

Preferred stock often must pass earnings and dividends tests. Wisconsin limits preferred stock to 5 percent of assets, Illinois 10 percent, New Jersey and New York each 2 percent. Some states restrict preferred stock within their leeway provisions and some express maximum holdings as a percentage of surplus. In New York, a LIC cannot own more than 20 percent of the outstanding preferred stock of a firm.

Common stock often must meet earnings or dividends requirements, and New York requires listing on a recognized exchange. New York quantitative restrictions were significantly liberalized in 1969; in New York, common stock investments now are limited to the lesser of 10 percent of assets or 100 percent of surplus (5 percent and 50 percent prior to 1969). The limit on holdings of one issuer is one percent of assets and five percent of the issuer's outstanding shares (0.2 percent and 2 percent before 1969). For purposes of complying with this law, stocks are valued at cost. Until 1969, virtually all other states had significantly less binding quantitative restrictions on common stock than New York. In 1970, limits were placed on real estate investment, raising the limits from five to ten percent of assets, and parcel sizes are also limited. Income-producing real estate and common equities have been the most restricted categories for life companies.

"Leeway" or "basket" clauses have been enacted by many states to permit LICs to invest a portion of their assets in a manner otherwise prohibited by their statutes. The leeway provision is 8 percent in Connecticut, 5 percent in Wisconsin and Pennsylvania, and 3.5 percent (2.0 percent before 1966) in New York, and 2 percent in New Jersey. In Massachusetts, all assets in excess of three-fourths of reserves may be invested freely. Some of the leeway clauses are qualified to prevent life companies from acquiring more than a given percentage of a corporation's outstanding common shares.

There are indications that some of the portfolio constraints listed above have severely constricted life insurance investment behavior. Perhaps, the best evidence that the restrictions have been binding is the persistent (and successful) lobbying of the industry for liberalization of these statutes. For example, permissible common stock holdings for New York companies were raised from zero to the lesser of 3 percent of assets or one-third of surplus in 1951. In 1957, the limit was raised to the lesser of 5 percent of assets or one-half surplus, and in 1969, it was raised to the lesser of 10 percent of assets or 100 percent of surplus. The limit on holding of a single corporation's common shares was raised from 0.1 percent to 0.2 percent to 1.0 percent. The maximum holding of a corporation's shares as a fraction of its outstanding shares has been raised from 2 to 5 percent. Dividend and earnings requirements on common shares were relaxed in 1967. Recently, statutes on investments outside of common stock have also been liberalized.

Another indication that some portfolio constraints have been binding is that companies in states with less stringent limits often take advantage of them. Of the largest 150 companies at the end of 1957, the thirteen New York companies held 0.6 percent of common stock (2.5 percent total stock); the ten Massachusetts companies held 7.1 percent of common stock (8.7 percent of total stock); and the ten Texas companies held 7.9 percent of common stock (9.7 percent total stock).[32] Some of the Massachusetts or Texas companies may have not been completely free of the New York laws if they were licensed to sell in New York and, consequently, had to comply substantially with its investment laws. British life companies have held more equities than American companies; they have increased their holdings of preferred and common shares from 18.7 percent of assets in 1950 to 22.2 percent in 1955 and to 26.3 percent in 1960. The fraction in common shares alone was 11.1, 15.3, and 21.2 percent for the same

---

[32] Brimmer, Life Insurance Companies, Table IX-12, p. 351.

three dates.[33] Both the actual fraction of common stock holdings and the fraction permitted under New York law have exhibited increasing trends in the post-war years. The interstate and international comparisons imply that New York constraints have been binding.

Other taxes and regulations

Numerous additional taxes and regulations besides those discussed earlier affect life insurance portfolio decisions. State and local governments impose premium taxes, property taxes, income taxes, unemployment taxes, license fees, and other miscellaneous levies on LICs. Taxes on life and health premiums average 2 percent of premiums and range from 1-3/4 to 4 percent.[34] Some states permit credits against premium taxes based on other state taxes paid or on investments made within the state; other states tax only out-of-state companies or tax domestic companies at lower rates. Retaliatory provisions are employed by various states, such as taxing out-of-state companies at the rate the foreign states would tax a domestic company or including a minimum rate and then charging domestic companies an even lower rate.

---

[33] G. Clayton and W. T. Osborn, Insurance Company Investment, Principles and Policies (London: George Allen and Unwin, Ltd., 1965). Appendix Table 2, p. 254.

[34] Dan M. McGill, Life Insurance (rev. ed.; Homewood, Illinois: Richard D. Irwin, Inc., 1967), p. 928.

The strongest geographical restriction on investment choices is the Texas law (the Robertson Law) requiring LICs to purchase Texas securities totaling at least 75 percent of the reserves behind policies sold in the state.[35] After the law was passed in 1907, most of the large companies left the state but gradually returned, perhaps because the strength of the Texas economy supplied sufficient securities to fulfill the 75 percent requirement. By 1959, nine states, in addition to Texas, gave tax reductions to LICs investing certain percentages of their assets within the state. The existence of absolute geographical restrictions or tax rebates based on geographical distribution of assets is inconsistent with the development of a national capital market. However, the premium taxes and other state and local taxes listed above do not significantly interfere with the allocation of investible funds, although there may be some minor influences.

Any regulations which affect the economic units selling primary securities in the capital markets or the institutions buying those primary securities might affect life insurance portfolio investment choices. For example (as mentioned previously), the Federal Power Commission and the Securities and Exchange Commission usually insist

---

[35]See Brimmer, Life Insurance Companies, pp. 66-7.

that utility bonds be callable immediately at low call premiums; as a result LICs are reluctant to acquire utility bonds when interest rates are considered above normal. Mortgage insurance and the attendant regulations of the characteristics of the insured mortgages by the Veterans Administration and the Federal Housing Administration alter the risk and return parameters of this type mortgage. Regulations on other financial intermediaries would also indirectly affect the life insurance industry's relative competitiveness as a supplier of loanable funds to the capital markets.

Summary

The impact of federal income taxation, valuation procedures, state qualitative and quantitative restrictions, and other taxes and regulations on life portfolios are very complex. The laws and regulations are so interrelated that the impact of only the bluntest can be singled out and measured. Nevertheless, portfolio decisions must be made and evaluated in this environment.

Life companies and their regulators coexist in a partially closed system in which commonly held options are reinforced and persist. The nature of the relationship between the industry and its regulators tends to shape investment policy and inhibit competitive responses. Jones speculates on this relationship as follows:

> Presumably life companies approval of statutory regulations is based in part on its usefulness as a device to restrict competition. Also, some companies no doubt genuinely fear management discretion with respect to investment policy would lead to speculative abuses by some small life companies, resulting in losses to their policyholders and bad public relations for the industry. This mix of motives is often present in regulated industries and it is difficult to gauge their relative importance.[36]

Walter also has reservations about the industry's performance.

> Whether life companies as a group perform as well as they might is doubtful. The common focus seems to be in matching or outperforming competition rather than exhausting the feasible possibilities for higher returns. In part, this orientation is attributable to the hesitancy of most life companies to deviate notably from standard patterns of behavior. In part, it is due to a tradition of limited investment staff and to certain ambiguities in the risk-taking capacity of life companies.[37]

The avoidance of an abnormal portfolio is fostered because the regulators can be expected to accommodate an industry-wide crisis, but not the crisis of an individual firm; thus, regulations appear to be fair-weather standards. Valuation procedures, tax laws, or other portfolio restrictions have responded to the problems of the industry as a whole during the 1918-19 influenze epidemic, the depression, and World War II. If the firm's crises do not coincide with those of industry, help from regulators will not necessarily be forthcoming. The penalty for a

---

[36]Jones, Investment Policies, p. 102.

[37]Walter, The Investment Process, pp. 6-7.

period of poor performance is ameliorated if the same problems are shared by your competitors. This is accomplished not by being better than the competition, but merely by being like them. In such a system, innovation and calculated risk-taking will be rare.

The following chapters present an analytical model of life insurance portfolio behavior and empirically test some hypotheses derived from that model. The sources of risk and the tax and regulatory environment guide the formulation of the anlaytical model, suggest some specific hypotheses which will be tested, and affect the quality of the data base used in the statistical analysis.

CHAPTER III

MODEL OF LIFE INSURANCE PORTFOLIO BEHAVIOR

Chapter III presents and analyzes a chance-constrained model of life insurance portfolio behavior which is used to generate hypotheses, some of which are statistically tested in Chapter IV. After the general model is introduced, two subsystems of the model, namely the investment opportunity subsystem and the preference subsystem, are explored in detail. In conclusion, possible extensions and limitations of the model are discussed, and the chance-constrained model is compared to other portfolio models.

A life insurance company's ability to satisfy its contractual obligations depends on its mortality experience, its future operating expenses, and its investment performance. This study concentrates on the third aspect mentioned above and abstracts away from uncertainties connected with the first two by assuming that the mortality of a company's policyholders is precisely that predicted by the company's mortality tables, that the company receives premiums from its policyholders on whole life policies until they die (at which time the company pays death benefits equal to the face value of the policies), and that the company has no operating expenses. Thus, the basic analysis makes no attempt to incorporate policy loans and surrender of policies for cash value. Subsequently, it will be shown how these assumptions can be relaxed and entered in the model.

The Chance-Constrained Model

The economic relationships concerning life insurance portfolio management can be summarized in this chance-constrained model:[1]

$$\max \quad \text{Return on assets owned at time } t \qquad (1)$$

$$\text{s.t.} \quad \text{Prob}[(1+r)\text{Assets}_t > \text{Obligations}_{t+1}] \geq \alpha \qquad (2)$$

other portfolio constraints

where α = probability of solvency

$(1-\alpha)$ = probability of insolvency

and r = expected rate of return on assets

The objective of the life company is to maximize its rate of return on assets subject to the constraint that it has a high probability of being solvent in the next period

---

[1]For examples of chance-constrained programming applied to other financial intermediaries, see Joel Fried, "Bank Portfolio Selection," Journal of Financial and Quantitative Analysis, V (June, 1970), 203-27; N. H. Agnew, R. A. Agnew, J. Rasmussen, and K. R. Smith, "An Application of Chance-Constrained Programming to Portfolio Selection in a Casualty Insurance Firm," Management Science, XV (June, 1969), B512-20; and A. Charnes and Sten Thore, "Planning for Liquidity in Financial Institutions: The Chance-Constrained Method," Journal of Finance, XXI (December, 1966), 649-74. The Charnes-Thore article is applied to savings and loan associations.

and subject to any other relevant portfolio constraints. This basic model analyzes the function of life insurance companies as financial intermediaries buying primary securities and issuing their own unique indirect security, policy reserves. This model can be developed more fully using the gross dollar amounts invested in various types of assets or using the proportions of assets invested in various types of securities as endogenous variables. Proportions are used here because relative sizes are easier to visualize than absolute sizes and because hypotheses stated as proportions prove more convenient to test in Chapter IV. The funds allocated to various types of securities are the instruments the LIC uses to maximize the value of its objective function.

Assume the rate of return on assets is multivariate normal. Then the above model can be rewritten more completely:

$$\max \quad r = R' X_t \tag{3}$$

$$\text{s.t.} \quad (1+R)' X_t - Z(X' V X)^{1/2} > LR_{t+1} - NCF_{t+1} \tag{4}$$

$$X_i \geq 0 \text{ for all securities} \tag{5}$$

$$X_i \leq K_i \text{ for some } i \tag{6}$$

$$\sum X_i = 1 \tag{7}$$

where:

X = vector of proportions of assets composed of n securities (n x 1)

R = vector of expected rates of returns on these n securities (n x 1)

Z = inverse of the standard normal cumulative distribution

V = variance-covariance matrix of rates of return on these securities ($n \times n$)

$(X'VX)^{1/2}$ = standard deviation of return of portfolio X

LR_{t+1} = (legal reserves in period $t+1$) / (assets in period t)

NCF_{t+1} = (net cash flow_{t+1}) / (assets_t)

= (premiums_{t+1} - death benefits_{t+1}) / assets_t

In addition, the following symbols will be used in the remainder of this section:

LR_t = (legal reserves_t) / assets_t

$S_t = 1 - LR_t$ = surplus_t / assets_t

k = contractually promised rate of return on reserves

$\sigma = (X'VX)^{1/2}$

The life company maximizes its rate of return (3) subject to a probabilistic solvency constraint (4), non-negativity constraints (5), legal constraints (6), and a balance sheet constraint (7). Obviously no general solution of this model is possible: the model must be solved algorithmically for a specific case.

The chance-constrained model can be compartmentalized into two subsystems: a preference set derived from constraint (4) and an opportunity set derived from constraints (5), (6), and (7). The LIC attempts to maximize its rate of return given the feasible set determined by the intersection of the preference set and the opportunity set.

Since $LR_{t+1} - NCF_{t+1} = (1+k)LR_t$,[2] the chance-constraint (4) can be rewritten:

$$(1+R)'X_t - (1+k)LR_t - Z(X'VX)^{1/2} > 0 \tag{8}$$

and since $S_t + R'X_t - kLR_t - Z(X'VX)^{1/2} > 0$ (9)

this can be rewritten as

$$S_t + r - kLR_t - Z\,\sigma > 0. \tag{10}$$

If the current surplus plus expected earnings less interest on reserves is positive, the company is solvent, and the company has a probability of α of being solvent if this expected result minus $Z\sigma$ is still positive. This constraint is linear in r and σ and can be rewritten

$$\sigma \leq \frac{1}{Z}(S_t - kLR_t) + \frac{1}{Z}r. \tag{11}$$

This equation, given Z, S_t, k, and LR_t, specifies the preference set and the line where the equality holds will be called the preference function.

Any vector X which satisfies the non-negativity, legal, and balance sheet constraints generates a particular $r = R'X$ and $\sigma = (X'VX)^{1/2}$. All possible combinations or r and σ satisfying these constraints constitute the opportunity set enclosed by DIEJFGH in Figure 3-1.

---

[2]This is established in a later section of this chapter.

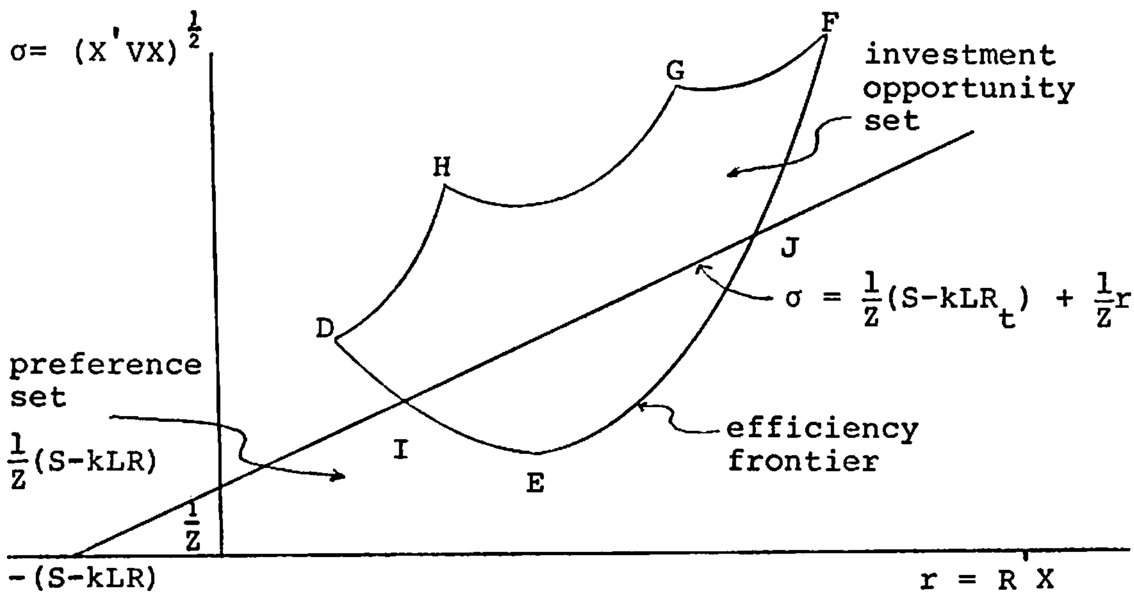


Fig. 3-1.--Chance-Constrained Portfolio Model

The preference set is also indicated on Figure 3-1 as the area below and including the preference function $\sigma = \frac{1}{Z}(S_t\text{-}kLR_t) + \frac{1}{Z}\,r$. The intersection of the opportunity set and the preference set is bounded by IEJ, and the rate of return is maximized at point J. Any point satisfying the chance-constrained model will fall along EJF, the Markowitz efficiency frontier, and will have the maximum return for a given level of risk (σ) and the minimum risk for a given level of return. In general, the optimal portfolio is the set of assets which yields the risk-return combination where the efficiency frontier and preference function intersect.

The next two sections of this chapter examine the investment opportunity subsystem and preference subsystem and the effects on portfolio choices of changes in the parameters of these two subsystems.

The Investment Opportunity Subsystem

The Markowitz model

The set of combinations of portfolio risk and return available to the life company depends on the parameters of the expected return vector R, the variance-covariance matrix V, the legal constraints on some types of securities, the non-negativity constraints, and the balance sheet constraint. Changes in any of these parameters would shift the opportunity set and, of course, result in a new portfolio where the preference function intersects the shifted efficiency frontier. Any parameter changes resulting in a rightward (downward) shift in the efficiency frontier will result in management's choosing a higher return, riskier portfolio along the preference function. Likewise, a leftward (upward) shift in the efficiency frontier will result in a less risky, lower return portfolio. The portfolio of securities which yields a particular point on the efficiency frontier can be found with this model:[3]

---

[3]The solution for this model without the non-negativity and legal constraints is found in Jack Francis and Stephen Archer, Portfolio Analysis (Englewood Cliffs, New Jersey: Prentice Hall, Inc., 1971), pp. 78-80. The basic model is found in Harry Markowitz, "Portfolio Selection," Journal of Finance, VII (March, 1952), 77-91, and Harry Markowitz, Portfolio Selection: Efficient Diversification of Investments (New York: John Wiley & Sons, Inc., 1959).

$$\begin{aligned} \text{min:}\quad & X'VX \\ \text{s.t.}\quad & R'X = r^* \\ & X_i \geq 0 \text{ for } i = 1, n \\ & X_i \leq K_i \text{ for some } i \\ & \textstyle\sum X = 1 \end{aligned}$$

where r^* is the desired rate of return on the portfolio and all other symbols have the same definitions as in the first section of this chapter. The portfolio found with this model is Markowitz efficient if and only if $\frac{d\sigma^2}{dr^*} > 0$. Life companies may have made different portfolio choices because the parameters in the above model change through time, the parameters are heterogeneous across companies at a point in time, and the perceptions of different managements, even given the same environment and information, may not be homogeneous.

For illustrative purposes, assume a LIC chooses among the following five securities:

1. bonds,
2. government insured mortgages,
3. conventional mortgages,
4. preferred stock, and,
5. common stock,

and that these securities have the following return vector and variance-covariance matrix:

$$R = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \end{bmatrix} = \begin{bmatrix} 4.0 \\ 5.0 \\ 5.5 \\ 6.0 \\ 10.0 \end{bmatrix} \quad V = \begin{bmatrix} 3.0 & 3.0984 & 3.0984 & 3.0984 & 5.4222 \\ 3.0984 & 5.0 & 4.0 & 4.0 & 7.0 \\ 3.0984 & 4.0 & 5.0 & 4.0 & 7.0 \\ 3.0984 & 4.0 & 4.0 & 5.0 & 7.0 \\ 5.4222 & 7.0 & 7.0 & 7.0 & 20.0 \end{bmatrix}$$

Assume the company has legal maximum proportions of assets on conventional mortgages of 40 percent, on preferred stock of 4 percent, and on common stock of 10 percent, and that the company wants a 5.5 percent return on assets. Since all three legal constraints are binding at this desired rate of return, the portfolio weights minimizing the variance for this return are found by minimizing this Lagrange function:

$$L = \sum_{i=1}^{5} \sum_{j=1}^{5} X_i X_j \sigma_{ij} + \lambda_1 \left(\sum_{i=1}^{5} X_i R_i - r^* \right)$$

$$+ \lambda_2 \left(\sum_{i=1}^{5} X_i - 1 \right) + \lambda_3 (X_3 - .40) + \lambda_4 (X_4 - .04)$$

$$+ \lambda_5 (X_5 - .10) \qquad (12)$$

The system of ten partial derivatives of (12) with respect to five X_i's and five λ's is given as follows:

$$\begin{bmatrix} 2\sigma_{11} & \cdot & \cdot & \cdot & 2\sigma_{15} & R_1 & 1 & 0 & 0 & 0 \\ \cdot & \cdot & & & \cdot & R_2 & 1 & 0 & 0 & 0 \\ \cdot & & \cdot & & \cdot & R_3 & 1 & 1 & 0 & 0 \\ \cdot & & & \cdot & \cdot & R_4 & 1 & 0 & 1 & 0 \\ 2\sigma_{51} & \cdot & \cdot & \cdot & 2\sigma_{55} & R_5 & 1 & 0 & 0 & 1 \\ R_1 & R_2 & R_3 & R_4 & R_5 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ r^* \\ 1 \\ .40 \\ .04 \\ .10 \end{bmatrix} \quad (13)$$

or $A \cdot B = C$, in matrix notation. (14)

Since A is square and nonsingular, its inverse exists. Pre-multiplying both sides of (14) by A^{-1} yields the solution vector

$$B = A^{-1}C \quad (15)$$

The first five elements of vector B are the proportions of total assets in each of the five securities.

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} = \begin{bmatrix} .24 \\ .22 \\ .40 \\ .04 \\ .10 \end{bmatrix} \begin{array}{l} \\ \\ \text{legal constraint} \\ \text{legal constraint} \\ \text{legal constraint} \end{array}$$

By holding the security proportions X, the LIC achieves the minimum possible variance, given its desired rate of return, non-negativity constraints, legal constraints, and balance sheet constraint. Of course, in this example, the non-negativity constraints were not binding. The portfolio has an

$r = 5.5\%$

$\sigma^2 = 4.491$, and

$\sigma = 2.119$.

This risk-return combination is one point on the efficiency frontier.

Sensitivity of model to parameter changes

The economic interpretation of the Lagrange multipliers is easily found by taking the appropriate derivatives of the Lagrange function (12). Let $\sum X_i = A$.

$$\frac{\delta L}{\delta r^*} = \frac{\delta \sigma^2}{\delta r^*} + \lambda_1 = 0 \qquad \lambda_1 = -\frac{\delta \sigma^2}{\delta r^*}$$

$$\frac{\delta L}{\delta A} = \frac{\delta \sigma^2}{\delta A} + \lambda_2 = 0 \qquad \lambda_2 = -\frac{\delta \sigma^2}{\delta A}$$

$$\frac{\delta L}{\delta X_3} = \frac{\delta \sigma^2}{\delta X_3} + \lambda_3 = 0 \qquad \lambda_3 = -\frac{\delta \sigma^2}{\delta X_3}$$

$$\frac{\delta L}{\delta X_4} = \frac{\delta \sigma^2}{\delta X_4} + \lambda_4 = 0 \qquad \lambda_4 = -\frac{\delta \sigma^2}{\delta X_4}$$

$$\frac{\delta L}{\delta X_5} = \frac{\delta \sigma^2}{\delta X_5} + \lambda_5 = 0 \qquad \lambda_5 = -\frac{\delta \sigma^2}{\delta X_5}$$

The Lagrange multipliers are the negative of the marginal effect on portfolio variance of a marginal change in each of five constraints.

$$\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \end{bmatrix} = \begin{bmatrix} -1.99 \\ 1.36 \\ .64 \\ 2.35 \\ 2.73 \end{bmatrix}$$

$\frac{\delta \sigma^2}{\delta r^*} = -\lambda_1 = 1.99$. The marginal change in the standard deviation is $\frac{\delta \sigma}{\delta r^*} = \frac{\delta \sigma^2}{\delta r^*} \frac{\delta \sigma}{\delta \sigma^2} = \frac{\delta \sigma^2}{\delta r^*} \frac{1}{2\sigma} = .47.$

The slope of the efficiency frontier at this level of return is 0.47, the necessary positive slope.

$\frac{\delta \sigma^2}{\delta A} = -\lambda_2 = -1.36$ shows the marginal change in variance which would occur if the amount of assets were to marginally exceed unity and all other constraints were unchanged. With more assets to achieve the same amount of return, the variance decreases. If no portfolio weights are constrained, λ_1 and λ_2 will be of opposite signs but they will not necessarily be of opposite signs if there are binding constraints on the portfolio proportions.

$\frac{\delta \sigma^2}{\delta X_3} = -\lambda_3 = -.64$, $\frac{\delta \sigma^2}{\delta X_4} = -\lambda_4 = -2.35$, and $\frac{\delta \sigma^2}{\delta X_5} = -\lambda_5 = -2.73$ are the shadow prices of the legal constraints on conventional mortgages, preferred stock, and common stock, respectively. In all three cases, a relaxation of the legal constraints would result in a portfolio with less variance for the desired rate of return.

In general, a relaxation of a binding legal constraint will shift the efficiency frontier downward (rightward), permitting portfolios with less variance for a given return and more return for a given variance. In the chance-constrained model, this shift of the efficiency frontier will result in the LICs choosing a riskier, higher return portfolio with the same probability of insolvency. Of course, relaxation of a binding legal constraint on a risky asset results in the LICs holding more of that asset.

Changes in the expected return of a security in the return vector or in an element in the variance-covariance matrix might shift the efficiency frontier as well. If the expected return increases (decreases) on a security which is positively held, the efficiency frontier will shift rightward (leftward). Of course, in the chance-constrained model, a rightward shift of the efficiency frontier will cause the life company to choose a higher return, riskier portfolio. With the variance-covariance matrix unchanged, the portfolio shifts caused by an increase in the expected return of a single security are complex. As the LIC moves to a riskier, higher return portfolio and if the changed return is for a high return security, the LIC would probably hold more of that security.

But, if the change in return is on a low return security, the LIC may use the increased rate of return from the low return security to offset the additional risk of investing more of its assets in riskier securities. The yield on a given security is not necessarily positively related to the demand for that security, ceteris paribus.[4] In the chance-constrained model, the life company's risk-taking capacity is positively related to its surplus and expected rate of return on total assets and negatively related to the rate it pays on its liabilities. If the portfolio weights were unchanged, an increase in the rate of return on one security would increase the portfolio rate of return and the variance would remain constant. However, as the LIC shifts to a higher variance portfolio, this may mean holding either more or less of the security whose yield increased.

A decrease (increase) in a variance or covariance parameter will shift the efficiency frontier downward (upward) if the change involves a security positively held, and will cause the LIC to choose a riskier, higher

---

[4]The condition where the demand for a security is negatively related to its yield has analogues in other areas of economic theory, such as the Giffin commodity, the backward-bending supply curve of labor, and the target savings hypothesis. This situation might exist if the yield on fixed income securities increased, ceteris paribus, and life insurance companies purchased less fixed income securities and more equities. A simple proof of this portfolio shift, as well as the other shifts discussed in the remainder of Chapter III, is given in Appendix III.

return portfolio. The demand for a particular security may be either positively or negatively related to its own variance or covariance with another security.

The impact of simultaneous changes in several, or perhaps all, parameters of the return vector and variance-covariance matrix could be found in addition to the partial effects of a single change. For example, all yields could be increased by a constant amount with the variance-covariance matrix constant, resulting in a parallel rightward shift of the investment opportunity set and efficiency frontier, as long as no constraints are operating. This, in turn, will cause the LIC to choose a riskier, higher return portfolio and to shift toward relatively riskier types of securities.

By decreasing the square roots of all the terms in the variance-covariance matrix by a constant proportion, with the return vector given, a second type of change would be effected which would result in a downward shift of the efficiency frontier. The LIC will, then, shift to a higher return, riskier portfolio and will shift toward the relatively more risky types of securities.

The assumption that relationships between risk and return are homogeneous of degree one could produce another possible set of changes. Each term of the return vector and variance-covariance matrix increases (or decreases) in the same proportion. Any risk-return combination in the opportunity set or on the efficiency frontier will be shifted (on a ray through the origin) a distance proportionally farther from the origin than the combination previously was located. The LIC will choose a higher risk, higher return portfolio, but since the preference function has a positive vertical intercept (and smaller slope than a ray from the origin to the intersection of the preference function and efficiency frontier), it will shift toward relatively less risky types of securities. Many further shifts are possible but will not be enumerated.

Relaxation of binding legal constraints will result in heavier investment in the affected types of securities. The portfolio shifts induced by changes in expected returns or variances or covariances depend upon the specific situation.

The Preference Subsystem

Risk-aversion and risk-taking capacity

The risk-return choices a LIC would be willing to consider depend on the amount of the company's surplus, its contractual obligations to policyholders, and management's degree of risk-aversion. Prior to evaluating the sensitivity of LICs' portfolio preferences to these variables, the function of portfolio risk and return to the LIC's obligation to its policyholders must be specified.

The economic obligation of a LIC to its ordinary life policyholders is easily established. At the time a policy is issued the premium A is found by discounting the stream of expected death benefits and the stream of expected premiums back at the discount rate k, setting the present value of expected death benefits equal to the present value of expected premiums, and solving for A.[5]

$$\sum_{t=0}^{n} \frac{m_t F}{(1+k)^t} - \sum_{t=0}^{n} \frac{A(1-\sum_{u=0}^{t} m_u)}{(1+k)^t} = 0 \qquad (16)$$

where:

m_t = conditional probability that a policyholder dies in period t, given that he was alive at t=0, the time the policy was issued. $m_o = 0$ and $\sum_{t=0}^{n} m_t = 1$.

F = face value of the policy. F is paid to the policyholder's estate at the end of the period in which he dies.

A = annual level premium paid at the beginning of each period by all living policyholders.

k = discount rate used to set premiums.

---

[5]See Robert Cissell and Helen Cissell, Mathematics of Finance (3rd ed.; Boston: Houghton Mifflin Company, 1969), pp. 280-82, and Floyd S. Harper and Lewis C. Workman, Fundamental Mathematics of Life Insurance (Homewood, Illinois: Richard D. Irwin, Inc., 1970), pp. 219-22.

NCF_t = expected net cash flow between the LIC and the policyholder at time t. NCF_t equals the expected premium less the expected death benefit at time t.

$m_t F$ = the expected death benefit on a policy at time t.

$(1 - \sum_{u=0}^{t} m_u)$ = the probability of the policyholder being alive at time t.

$A \cdot (1 - \sum_{u=0}^{t} m_u)$ = the expected premium payment at time t.

After the policy is in effect, the equality between the present value of expected death benefits and the present value of expected premiums is disturbed. At $0 < s < n$,

$$\sum_{t=s+1}^{n} \frac{m_t F}{(1+k)^t} - \sum_{t=s+1}^{n} \frac{A(1 - \sum_{u=0}^{t} m_u)}{(1+k)^t} =$$

$$- \sum_{t=0}^{s} \frac{m_t F}{(1+k)^t} + \sum_{t=0}^{s} \frac{A(1 - \sum_{u=0}^{t} m_u)}{(1+k)^t} \tag{17}$$

Multiplying both sides of the expression by $(1+k)^s$ yields[6]

[6]See Cissell, pp. 289-93, and Harper, pp. 244-51.

$$\sum_{t=s+1}^{n} \frac{m_t F}{(1+k)^{t-s}} - \sum_{t=s+1}^{n} \frac{A(1-\sum_{u=0}^{t} m_u)}{(1+k)^{t-s}} =$$

$$\sum_{t=0}^{s} [A(1-\sum_{u=0}^{t} m_u) - m_t F] \; (1+k)^{s-t} \tag{18}$$

Obviously, at time s, the present value of future death benefits minus the present value of future premiums equals the sum of all net cash flows up to and including time s compounded to time s at the rate k. The deficiency of the present value of future death benefits is the life company's legal reserves, its contractual obligations resulting from its life insurance policies. The legal reserves seen in equation (18) also equal all previous net cash flows compounded forward to the present. If these net cash flows were invested in assets which earned a rate of return greater than k, the company would have assets exceeding its liabilities. If the assets earned less than k, the company's surplus would be diminishing, perhaps becoming negative (the company is insolvent).

At any time s, the company's obligations to its policyholders in the next period will be (1+k) times its current legal reserves. This is not to say that legal reserves at time s + 1 are 1 + k times current legal reserves, since additional premiums will be received and death benefits disbursed. In period s + 1, $LR_{s+1} - NCF_{s+1} = (1+k)LR_s$.[7] If the company's assets in the next period exceed its obligations in that period, the company remains solvent. The life insurance portfolio enables the company to fulfill its obligations to its policyholders by investing the net cash flows from policyholders at a return of at least that rate used to set the premiums on the policies. Thus, having established the relationship between the life company's obligations to its policyholders and solvency, the relationships among the variables S(LR), α(Z), k, r, or σ and solvency are clear.

---

[7]This is easily established since

$$LR_s = \sum_{t=s+1}^{n} \frac{m_t F}{(1+k)^{t-s}} - \sum_{t=s+1}^{n} \frac{A(1-\sum_{u=0}^{t} m_u)}{(1+k)^{t-s}}$$

$$LR_{s+1} = \sum_{t=s+2}^{n} \frac{m_t F}{(1+k)^{t-(s+1)}} - \sum_{t=s+2}^{n} \frac{A(1-\sum_{u=0}^{t} m_u)}{(1+k)^{t-(s+1)}}$$

$$LR_s(1+k) = \sum_{t=s+1}^{n} \frac{m_t F}{(1+k)^{t-(s+1)}} - \sum_{t=s+1}^{n} \frac{A(1-\sum_{u=0}^{t} m_u)}{(1+k)^{t-(s+1)}}$$

$$LR_s(1+k) - LR_{s+1} = \frac{m_{s+1}F}{(1+k)^{o}} - \frac{A(1-\sum_{u=0}^{t} m_u)}{(1+k)^{o}} = -NCF_{s+1}$$

and $LR_{s+1} - NCF_{s+1} = LR_s(1+k)$

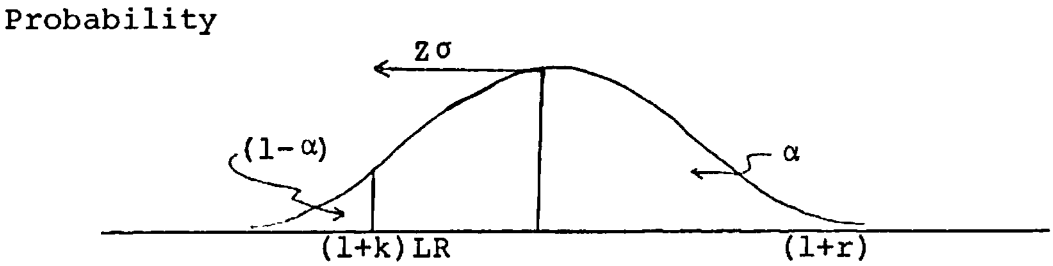


Fig. 3-2.--Probability Distribution of $\frac{\text{Assets}_{t+1}}{\text{Assets}_t}$

To the left of the point (1+k)LR = (1+k)(1-S) in Figure 3-2, the life company is insolvent. If r is normally distributed and (1-α) is the probability of insolvency management is willing to risk, the probability distribution of Figure 3-2 is a hypothetical distribution of (1+r) such that the probability of failure is exactly (1-α). This same probability of failure will exist for any distribution with a (1+r) exactly Zσ greater than (1+k)LR. Any particular risk-return combination where this is true corresponds to a point on the preference function $\sigma = \frac{1}{Z}(S-kLR_t) + \frac{1}{Z} r$ in Figure 3-1.

Sensitivity of the model to parameter changes

Given the opportunity set and corresponding efficiency frontier in Figure 3-1, it is easy to see the effect on the optimal portfolio J, if the parameters in the preference set changed. An increase in S or a decrease in k causes a parallel upward shift in

$\sigma = \frac{1}{Z}(S - kLR_t) + \frac{1}{Z} r$ such that the LIC maximizes its rate of return subject to its constraints at a new σ - r combination on the efficiency frontier with a higher return than at point J. A decrease in α (management's preference for solvency) decreases Z and the slope of $\sigma = \frac{1}{Z}(1 - kLR_t) + \frac{1}{Z} r$ becomes more positive, rotating through the point (-S + kLR, 0). A decrease in α would also result in a portfolio with higher risk and return. As long as the frontier of the preference set intersects the efficiency frontier EF (where $\frac{d\sigma}{dr} > \frac{1}{Z}$):

$$\frac{d\sigma}{dS} > 0 \qquad \frac{dr}{dS} > 0$$

$$\frac{d\sigma}{dk} < 0 \qquad \frac{dr}{dk} < 0$$

$$\frac{d\sigma}{d\alpha} < 0 \qquad \frac{dr}{d\alpha} < 0$$

of course,

$$\frac{d\sigma}{dLR} < 0 \qquad \frac{dr}{dLR} < 0 \quad \text{since } LR = 1 - S$$

and

$$\frac{d\sigma}{d(1-\alpha)} > 0 \qquad \frac{dr}{d(1-\alpha)} > 0 .$$

An increase in the current surplus, *ceteris paribus*, means that current assets are higher relative to the life company's obligations in the next period. Consequently, the life company can invest in a slightly riskier, higher return portfolio and have the same probability of remaining solvent. An increase in the rate a life company pays on its reserves (k), *ceteris paribus*, raises its obligations in the next period compared to current assets. The life company would shift to a slightly less risky, lower return portfolio to maintain the same probability of solvency. If management wishes to raise the probability of solvency (lower the probability of insolvency), *ceteris paribus*, it can do so by shifting to a less risky, lower return portfolio. These conclusions necessarily follow whenever the preference function intersects the efficiency frontier, providing the slope of the efficiency frontier is greater than the slope of the preference function if there are two intersections.

If $\sigma = \frac{1}{Z}(S - kLR_t) + \frac{1}{Z}r$ does not intersect the efficiency frontier but falls below it in Figure 3-1, the intersection of the preference set and the opportunity set is the null set. This result is, of course, absurd for a life company holding assets. Management will lower the probability of solvency until a real solution is obtainable. If $\sigma = \frac{1}{Z}(S - kLR_t) + \frac{1}{Z}r$ passes above the efficiency frontier, management will choose the highest return portfolio in the opportunity set, which is the highest return portfolio on the efficiency frontier (point F). In this case, minor changes in the parameters of the preference function will not alter the portfolio until the parameters of the preference function intersect the efficiency frontier below F.

Limitations and Extensions of the Chance-Constrained Model

Any model can be criticized on the quality of the parameters in the model, on the particular endogenous and exogenous variables included or excluded from the model, and on the appropriateness of the model compared to alternative models.

Limitations of the model

In this chapter, LICs are presumed to possess assets of a given market value which they can currently apportion, subject to constraints, such that the market value of these assets in the next period has a high, subjectively chosen probability of exceeding the company's obligations at that time. The portfolio choices made by management within this model depend on their perceptions of the relevant parameters. Faced with the same set of information, managements of different LICs will not have homogeneous *ex ante* expectations of expected yields and variances and covariances among yields. Likewise, while a legal limit may be clearly specified, management may consider an operational constraint either less than or greater than the legal standard. In addition, the measurement of surplus may not be uniform across companies.

Heterogeneity in the perceptions of parameters will lead to more variety in LIC portfolios.

Possible extensions of the model

The basic model developed in the first three sections of this chapter can be expanded by adding additional constraints.

Policy loans were not included in the model since they are made at the discretion of the policyholders, not the management; nevertheless, they can be included by adding them as an additional constraint. Hypothesizing that policy loans are a given quantity, one could add the necessary return and variance-covariance parameters, and then have management maximize its rate of return subject to this and other constraints.

As mentioned in Chapter II, LICs as a whole do not appear to have had liquidity problems, even in the face of severe general economic crisis; therefore, liquidity constraints were not considered in the model. If relevant, however, an additional deterministic or probabilistic liquidity constraint could be included.[8] Management may specify a level of liquid funds it wishes to exceed a high, subjectively chosen fraction of the

---

[8]For an example of a liquidity chance-constraint applied to commercial banks, see Joel Fried, "Bank Portfolio Selection," Journal of Finance and Quantitative Analysis, V, No. 2 (June, 1970), 203-27.

time. These liquid funds would be the cash realizable by liquidating the portfolio in the next period and would constitute its existing assets plus the expected appreciation less the costs of liquidating the portfolio. The level of liquid funds the company wishes in the next period will be chosen by management to reflect its projected cash needs. Since regulations tend to force LICs to buy a fairly conservative, marketable portfolio, the addition of a liquidity constraint on top of legal constraints could be redundant.

Several other important variables are easily incorporated into the model. Operating expenses can be included by considering them a cash outflow in the next period; loads on policies result in a larger cash inflow in future periods. A favorable (unfavorable) mortality experience causes a lower (higher) cash outflow for death benefits and a higher (lower) cash inflow from policyholder premiums in the next period. Cash surrenders and increases in policy loans do not alter the basic objective of maximizing the value of assets, subject to constraints, but would affect the level of liquid funds desired by management in a subsequent period.

As demonstrated by the preceding discussion, the chance-constrained model developed here is sufficiently general that additional variables and other considerations can be introduced by adding additional constraints. This model is useful because of its direct empirical content and should be compared with alternative single-period or multiple-period portfolio models.

<u>Comparisons with other portfolio models</u>

One alternative single-period model to the chance-constrained model developed here is the Markowitz model which defines an infinite set of efficient portfolios along the efficiency frontier. The tangency between the efficiency frontier and the highest indifference curve (defined in mean-variance space) finds the optimal portfolio which maximizes the economic unit's utility. The basic difficulty is empirically relating the economic variables relevant for LIC portfolio decisions to the parameters of the utility function. While discussing a similar problem, one writer says

> However, the generality which made the model attractive for theoretical purposes may make it next to useless for practical application. It is hard to imagine a business consultant beginning his work by asking the company president to state his preferences over a set of stochastic processes. Should this happen, the president could, with some justification, reply that he does not know how such preferences could be expressed in a consistent manner, and he might also demonstrate that he knows very well how to express his opinion of certain consultants.[9]

---

[9]Karl Henrik Borch, <u>The Economics of Uncertainty</u> (Princeton, New Jersey: Princeton University Press, 1968), p. 181.

The measurability of most of the parameters of the chance-constrained model makes it attractive for empirical work.

Another single-period model to which the chance-constrained model can be compared is Baumol's expected gain-confidence limit model.[10] Baumol defines an efficient portfolio as one where there is a tradeoff between expected return (E) and the lower confidence limit (L) associated with that return. $L = E - k\sigma$, k subjectively determined. $\frac{dL}{dE} = 1 - k\,\frac{d\sigma}{dE}$ when $\frac{d\sigma}{dE} > \frac{1}{k}$. The Baumol (E,L) criterion excludes all portfolios on the Markowitz (E,σ) efficiency frontier below the point where $\frac{d\sigma}{dE} = \frac{1}{k}$. A utility function would have to be defined to choose among the portfolios which are Baumol (E,L) efficient. An expected gain-confidence limit model presents the same basic problem that the Markowitz model presents: empirically relating life insurance company parameters to a utility function.

A third single-period model which could have been used assumes that the economic unit can borrow and lend at a constant risk-free rate. The optimal portfolio is that portfolio on the efficiency frontier which is at the tangency between the efficiency frontier and a ray from

---

[10] William J. Baumol, "An Expected Gain-Confidence Limit Criterion for Portfolio Selection," Management Science, X (October, 1963), 174-82.

the risk-free rate.[11] In Figure 3-3, r_F is the risk-free rate, r_A and σ_A are the return and risk on assets, and r_S and σ_S are the return and risk on surplus due to borrowing at the risk-free rate r_F.

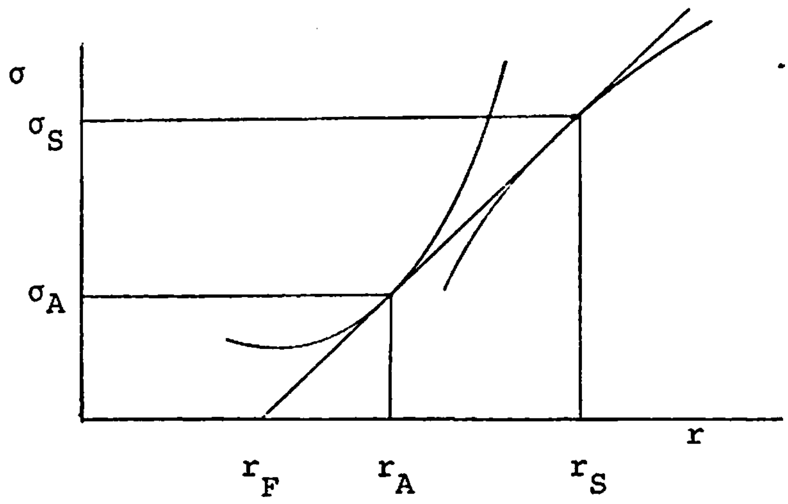


Fig. 3-3.--Sharpe-Lintner Portfolio Model

The economic unit borrows or lends at this risk-free rate and maximizes its utility where an indifference curve is tangent to the ray instead of where the indifference curve is tangent to the efficiency frontier. Increases in the risk-free rate, ceteris paribus, cause the economic unit to choose a riskier higher return portfolio and, conversely, decreases in the risk-free rate cause it

[11] William F. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," Journal of Finance, XIX (September, 1964), 425-42, and John Lintner, "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," Review of Economics and Statistics, XLVII (February, 1965), 13-37.

to choose a less risky, lower return portfolio. In this model, changes in the debt/equity or debt/asset ratios have no impact on the composition of the optimal portfolio; in the chance-constrained model, however, changes in the relative amount of surplus, *ceteris paribus*, result in the choice of a different portfolio. In the Sharpe-Lintner model, an increase in the risk-free rate would result in the choice of a riskier portfolio; this is the opposite of the shift which occurs in the chance-constrained model. With a positive risk-free rate, the portfolio shift prescribed by the Sharpe-Lintner model increases the probability of insolvency.

Without specifying the utility function, the concept of stochastic dominance can be applied to the same parameter shifts which have been examined in this chapter. Second-degree stochastic dominance applies to all concave (risk-averse) utility functions.[12] If the integral of the cumulative density function of the random variable (rates of return) for one portfolio choice is less than the same integral for another choice over the interval from the lowest possible return to any other arbitrary value, then the former choice is preferred by any risk-averse economic unit. If a change occurs which shifts the cumulative density function of possible returns such that the

---

[12]Josef Hadar and William R. Russell, "Rules for Ordering Uncertain Prospects," *American Economic Review*, LIX (March, 1969), 25-34.

cumulative probability of any arbitrary return is less than the previous cumulative probability for the same return, then this change allows the economic unit to move to a higher level of utility. It follows that a relaxation of a binding legal constraint or an increase in the yield on a security positively held would allow a life insurance company to move to a higher level of utility and a higher expected rate of return. The concept of stochastic dominance is sufficiently general that the portfolio shifts accompanying parameter changes are not necessarily determinable and the rate paid on reserves and the amount of surplus cannot explicitly be included in the analysis.

The life insurance investment process could be described with a multiple-period model instead of a single-period one since the portfolio problems are, in fact, dynamic.[13] In addition to the parameters of the single-period model, it would be necessary to make many assumptions, such as correlations among security yields between time periods, dividends, surrenders, policy loan extensions and repayments. The use of a multiple-period model instead of a single-period model would greatly magnify

---

[13]See Jan Mossin, "Optimal Multiperiod Portfolio Policies," The Journal of Business, XLI (April, 1968), 215-29, and Nils H. Hakansson, "Multi-period Mean-Variance Analysis: Toward a General Theory of Portfolio Choice," Journal of Finance, XXVI (September, 1971), 857-84.

the parameter estimation problem since *ex ante* yields and covariances might be needed for any two points in time within the time horizon used. A dynamic model might be so simple that it has no clear advantage over a one-period model, or so complex that it does not generate easily testable hypotheses. Since the purpose of this theoretical model is to provide empirically testable hypotheses, it is unclear that a more sophisticated analytical model would provide more useful hypotheses.

Summary

The chance-constrained model suggests several relationships between the portfolios chosen by LICs and other economic variables. The following situations should result in a shift toward greater investment in riskier securities: an increase in surplus, a decrease in the rate promised on legal reserves, an increase in management's willingness to risk insolvency, and a relaxation of a binding legal constraint on a risky asset. While no general statement of the effect of changes in expected returns or risks of particular securities is possible, a statistical analysis of the impact of changes in yield differentials and levels on portfolio selections is easily accomplished. An econometric study of some of these suggested economic relationships is the subject of Chapter IV.

CHAPTER IV

EMPIRICAL ANALYSIS OF LIFE INSURANCE PORTFOLIO BEHAVIOR

The chance-constrained portfolio model suggests several hypotheses about life insurance portfolio behavior which are examined in this chapter. To this end, Chapter IV explains the selection of the sample of life insurance companies, defines the relevant variables for analysis, explains the specific hypotheses to be tested, and presents a variety of cross-sectional/time-series regression results necessary to test these hypotheses. A simple ordinary least squares model supplemented by a least-squares dummy-variable model is used to study the role of the amount of surplus, the location of a company's charter, and the general level of financial yields. Next, these basic results are extended by considering the effects of relative yields and company size on portfolio choices. Also, tests of homogeneity between U. S. and Canadian companies are performed.

A variety of statistical techniques are employed in addition to the static ordinary least squares and least-squares dummy-variable models. A first-order autoregressive scheme is integrated into the least-squares dummy-variable model to moderate simultaneously for the effects of autocorrelation and interfirm variation. Both the Koyck and

Almon distributed lag techniques are used to study the dynamics of portfolio adjustments to changes in the independent variables. The Koyck model is estimated in a form which adjusts for autocorrelation since, in this case, the autocorrelation would cause biased results. The Almon model is particularly interesting here since it permits different distributed lags for each independent variable. These econometric techniques are the basis for several conclusions about the behavior of life insurance investments and the performance of the industry as a whole; in addition, these results combine with the theoretical analysis of Chapter III to give significant implications for financial econometric research in other areas.

The Companies in the Sample

The basis for the statistical tests in a pooled, cross-sectional/time-series sample of 104 large United States and Canadian life insurance companies possessing complete annual data for 1957-71.[1] The selection procedure was as follows:

1. All companies in the 1972 volume of Best's Insurance Reports: Life-Health Edition with assets exceeding $190.0 million were isolated. This set contained 130 companies ranging in size from $31.16 billion (Prudential) to

---

[1]See Appendices IV-1, IV-2, and IV-3 for a list of the relevant companies.

$191.48 million (Colonial).

2. Thirteen companies lacking complete data going back to 1957 were dropped from the sample, thus reducing the sample to 117 companies.

3. Thirteen more companies with assets less than $75.0 million on December 31, 1957, were dropped from the sample, reducing it to 104 companies.

The statistical analysis focuses on the largest companies for two simple reasons: an analysis of the large companies has more significance for aggregate industry behavior and, in addition, the smaller companies exhibit wider variances in portfolio behavior, perhaps because they are too small to diversify economically and because they have more limited managerial resources.

This sample includes 12 companies with Canadian charters and 92 with charters from various U. S. states. Nine companies are chartered in New York state and 41 companies are licensed to sell insurance in New York (63 are not). Fifty-eight of the companies are stock companies and 46 are mutuals. Since there were 1805 companies in the U. S. and 81 in Canada in 1971,[2] this sample includes only about 5.5 percent of the universe of companies from which the sample is drawn. However, as can be seen in Table 4-1, these 104 companies hold 91.60 percent of all assets held by the U. S. and

---

[2]Institute of Life Insurance, 1972 Life Insurance Fact Book (New York: Institute of Life Insurance, 1972), p. 87 and p. 103.

TABLE 4-1

INVESTMENTS OF 104 U.S. AND CANADIAN COMPANIES, DECEMBER 31, 1971

	(1) All U.S. and Canadian Cos.	(2) Proportion of Total Assets	(3) Sample of 104 Companies	(4) Proportion of Total Assets For Sample
Bonds	95,744*	.4119**	83,699*	.3931**
Mortgages	82,275	.3540	75,766	.3559
Bonds and Mortgages	178,019	.7659	159,465	.7490
Common Stock	N/A	N/A	9,507	.0447
Preferred Stock	N/A	N/A	3,304	.0155
Total Stock	16,005	.0689	12,830	.0603
Real Estate	7,780	.0335	6,825	.0321
Policy Loans	18,252	.0785	16,460	.0773
Total Assets	232,418		212,900	

Source: Institute of Life Insurance, 1972 Life Insurance Fact Book. (New York: Institute of Life Insurance, 1972).

*Millions of dollars. Column does not sum to total assets because of omission of miscellaneous assets.

**Column does not sum to unity because of omission of miscellaneous assets.

Canadian life companies. (The 117 companies remaining after step 2 held \$215.960 billion in assets; this was 92.92 percent of all assets held by U. S. and Canadian LICs. The comparable figures for the 130 companies in step 1 were \$221.190 billion and 95.17 percent).

Most of the statistical analysis of this chapter will be performed on the set of 92 large U. S. companies, thus excluding the 12 Canadian companies. These firms include 52 stock companies and 40 mutuals; 9 companies with New York charters and 83 companies chartered in other states; 39 companies licensed to sell in New York and 53 companies without this license. As seen in Table 4-2, these 92 companies held 91.85 percent of all assets held by U. S. life companies on December 31, 1971.

Definitions of Variables and Theoretical Implications for Coefficient Signs

Having described the sample, this section defines the relevant variables and outlines the hypotheses to be tested. The six dependent variables to be analyzed are:

$\frac{B}{TA}$ = bonds/total assets

$\frac{M}{TA}$ = mortgages/total assets

$\frac{F}{TA} = \frac{B}{TA} + \frac{M}{TA}$ = fixed assets/total assets

$\frac{CS}{TA}$ = common stock/total assets

TABLE 4-2

INVESTMENTS OF 92 U.S. COMPANIES, DECEMBER 31, 1971

	(1) All U.S. Companies	(2) Proportion of Total Assets	(3) Sample of 92 Companies	(4) Proportion of Total Assets For Sample
Bonds	89,435*	.4168**	78,411*	.3978**
Mortgages	75,409	.3514	69,774	.3540
Bonds and Mortgages	164,844	.7682	148,185	.7518
Common Stock	10,363	.0483	8,450	.0429
Preferred Stock	3,745	.0175	3,108	.0158
Total Stock	14,108	.0657	11,568	.0587
Real Estate	6,880	.0321	6,006	.0305
Policy Loans	17,065	.0795	15,385	.0781
Total Assets	214,579		197,090	

Source: Institute of Life Insurance, 1972 Life Insurance Fact Book. (New York: Institute of Life Insurance, 1972).

*Millions of dollars. Column does not sum to total assets because of omission of miscellaneous assets.

**Column does not sum to unity because of omission of miscellaneous assets.

$\frac{PS}{TA}$ = preferred stock/total assets

$\frac{TS}{TA} = \frac{CS}{TA} + \frac{PS}{TA}$ = total stock/total assets

These dependent variables are the proportion of total assets invested in different types of securities. Further disagregation was not fruitful, perhaps because Markowitz-type diversification is not significant in the face of extremely high correlations among yields on fixed income securities. Bonds and mortgages may be such nearly perfect (Markowitz) substitutes that the sum $\frac{F}{TA}$ can be predicted more accurately than either of its components. The dependent variables were chosen as proportions instead of dollars because of the presence of heteroskedasticity: if the dependent variables were measured in dollars instead of proportions, the error terms would systematically be larger for large companies than for small, resulting in inefficient estimators.

The following explanatory variables are considered:

$\frac{R}{TA}$ = (surplus + security valuation reserve)/total assets

YC = yield on corporate bonds

YM = yield on mortgages

YP = yield on preferred stock

SD = 1 if company is a stock company
= 0 if company is a mutual

NL = 1 if company is licensed to sell insurance in New York

= 0 otherwise

NC = 1 if company is chartered in New York

= 0 otherwise

TA = total assets in millions of dollars

The chance-constrained portfolio model generates several testable hypotheses about the relationship between the independent variables and the dependent variables which are summarized in Table 4-3.

The signs in the cells of Table 4-3 indicate whether a direct (+) or inverse (-) relationship is expected between two variables in the chance-constrained model. The predicted signs are not advanced as a universal case for any parameters inserted in the chance-constrained model, but as signs which could be predicted based on assumptions about the modern economic and institutional environment in which life insurance companies make their decisions. Of the dependent variables, it is assumed here that common and preferred equities are riskier investments than bonds and mortgages. The following discussion explains how each of the independent variables should effect the demand for the various asset groups used as dependent variables. Subsequently, econometric techniques are used to test these hypotheses about the relationships summarized in Table 4-3.

Since the relative quantity of surplus is positively related to the risk-taking capacity of the life insurance company, the higher $\frac{R}{TA}$ is, the more equities and less fixed income securities the company should hold. With more surplus,

TABLE 4-3

SIGNS OF REGRESSION COEFFICIENTS PREDICTED BY CHANCE-CONSTRAINED MODEL

	Dependent Variable					
Independent Variables	$\frac{B}{TA}$	$\frac{M}{TA}$	$\frac{F}{TA}$	$\frac{CS}{TA}$	$\frac{PS}{TA}$	$\frac{TS}{TA}$
$\frac{R}{TA}$	-	-	-	+	+	+
YC	-	-	-	+	+	+
$\frac{YM}{TA}$	-	+				
$\frac{YP}{YC}$					+	
SD			+	-	-	-
NL			+	-	-	-
NC			+	-	-	-
TA						

the company can invest more aggressively in riskier types of assets and maintain at least the same probability of solvency.

Yields are treated in two ways. First, the corporate bond yield is treated as a proxy for the *ex ante* yields of all financial assets since all yields are assumed highly correlated. An increase in the level of all yields (seen through an increase in YC) should cause a marginal shift away from fixed-income securities and towards riskier, higher-yielding equities. Second, relative yields expressed as ratios are used to explain portfolio shifts. In the chance-constrained model, the demand for a security is not necessarily positively related to its own yield.[3] However, if bonds and mortgages are assumed to be substitutes, the coefficient of $\frac{YM}{YC}$ should be negative in the bond equations and positive in the mortgage equations. Finally, if the demand for preferred stock is sensitive to relative yields, the coefficient of $\frac{YP}{YC}$ should be positive in the preferred stock equations.

Although the chance-constrained model contains no information about how different legal forms of organization

---

[3] It is interesting to note that William L. Silber assummed *a priori* that the demand for a security is positively related to its own yield. The agreement of signs with these expectations was one criterion Silber used to choose among alternative structural equations. See his *Portfolio Behavior of Financial Institutions: An Empirical Study with Implications for Monetary Policy, Interest-Rate Determination, and Financial Model Building* (New York: Holt, Rinehart and Winston, Inc., 1970) and "Portfolio Substitutability, Regulations, and Monetary Policy," *Quarterly Journal of Economics*, LXXXIII (May, 1969), 196-219.

should behave, two significant interpretations can be given to the signs of the stock dummy in a regression. Given the other independent variables, positive (negative) coefficients of the stock dummy in equity equations and negative (positive) coefficients in an equation for total fixed assets would infer that stock life insurance companies were less (more) risk averse than mutuals. A second interpretation of the stock dummy is based on a general difference between the life insurance products sold by stock and mutual companies. Life insurance policies sold by stock companies generally have lower premiums and higher assumed rates of return on policy reserves than policies issued by mutual companies. As established in Chapter III, this difference would cause stock companies to invest in a lower return, less risky portfolio, ceteris paribus. If future investment and mortality experience warrants it, the stock company will pay dividends to its stockholders and the mutual will pay dividends to its policy-holders. However, a stock company with lower premium rates and a higher assumed rate of return must invest more in fixed income securities and less in equities, given its degree of risk aversion and surplus. This second interpretation of the stock dummy provides an economic basis for predicting more conservative portfolios for stock companies.

Historically, New York has possessed the tightest regulations on portfolio choices of life companies, a situation which may affect the coefficients of the New York license or charter dummies. If the portfolio limitations are binding and companies licensed to sell in New York are regulated as tightly as those chartered there, the coefficients of NL should be significantly negative for the affected types of securities and the coefficients of NC should not be significant, since companies chartered in New York are a subset of those licensed to sell in that state. If the regulations are not equally stringent on both sets of companies, their impact on New York licensed companies chartered elsewhere is the coefficient of NL and their impact on New York chartered companies is the sum of the coefficients of NL and NC. If these limitations have been binding in the chance-constrained model, New York regulated companies should possess lower proportions of equities and, perhaps, conventional mortgages than LICs immune to New York laws, <u>ceteris paribus</u>. Hence, if the legal constraints are binding, the coefficients of NL and/or NC should be negative in regressions having $\frac{CS}{TA}$, $\frac{PS}{TA}$, and $\frac{TS}{TA}$ as dependent variables. The effect of the constraint on mortgages is ambiguous because FHA and VA mortgages (which are not restricted) may substitute for conventional mortgages and because of regional biases toward mortgages. If the predicted signs on the dummies do not appear, this would infer that the New York constraints on that asset were not binding.

Total assets (or some transformation of TA) can be included as an independent variable in order to measure the effect of company size on the demand for different types of securities. This coefficient would detect differences in the behavior of the large or small companies. Of course, the chance-constrained model does not predict the signs of the coefficients of size variables since the model neglects the scale of life companies.

Statistical Techniques

At this point it should be useful to outline briefly the specific econometric techniques used in this chapter to test hypotheses about the economic behavior of life insurance portfolios.

The most elementary models are ordinary least squares (OLS) regressions which attempt to explain the portfolio choices among the six dependent variables (bonds, mortgages, total bonds and mortgages, common stock, preferred stock, and total stock) as functions of the relative amount of surplus, the overall yield level, and dummies for the stock form of organization, a New York license, and a New York charter. Because much of the total variation of each dependent variable is interfirm variance, a least-squares dummy-variable (LSDV) model, which utilizes company dummies to eliminate between firm variation, is adopted subsequently. In the LSDV model, the stock, NY license, and NY charter dummies must be dropped since these dummies are linearly dependent with the company dummies. Aitkens generalized least squares estimators provide a potentially more efficient method of pooling cross-sectional/time-series data than either the OLS or LSDV approaches, but generalized least squares is not used here since, as will be shown below, the LSDV model should approximate closely the efficiency of the generalized least squares estimates.

The basic OLS or LSDV models are extended to consider some relationships beyond those initially considered. Yield ratios are used as independent variables to find if relative yields affect the demand for different classes of securities. Then, the size of the life insurance company is included to measure the impact of the scale of the company on its portfolio choices. Finally, an analysis of covariance test is used to test the hypothesis that U. S. and Canadian companies are a homogeneous set with respect to the equations estimated in this study.

Because of the presence of positive serial correlation, even in the LSDV model, the Hildreth-Lu scanning technique is used to adjust for first-order autocorrelation. While this procedure does not alter substantially the results of the empirical tests, it does provide additional insight into the behavior of the life insurance companies.

Two distributed lag models are estimated to explore the dynamics of the investment behavior. The Koyck distributed lag, which assumes that the impact of the independent variables declines exponentially with time, is found with an estimating equation which removes autocorrelation because the presence of serial correlation leads to inconsistent estimators using OLS estimates. In contrast, the presence of autocorrelation does not lead to inconsistent estimators when using the Almon lag technique. The assumption in the Almon technique, that the weights of the current and lagged independent variables lie on a polynomial, provides for much more efficient tests of the impact of these variables, subject, of course, to the risk of committing a specification error. The Almon model possesses two important advantages over the Koyck lag: it permits a more general shape of distributed lags and, importantly, it permits a different lag structure for each independent variable. This technique provides significant results about the dynamic impact of the independent variables.

Each of the above models is explained more completely in the following sections where it is used. In some cases, statistical results which add little to the overall results but should be included for logical completeness are relegated to appendices.

Cross-Sectional/Time-Series Results: Basic Models

The basic multiple regression results for the six dependent variables estimated as a function of $\frac{R}{TA}$, YC, the dummies SD, NL, and NC (or, alternately, dummies for each company) are presented in this section for 92 large U. S. companies covering the 1957-71 time period. The OLS model, presented first, yields significant coefficients for $\frac{R}{TA}$, YC, SD, NL, and NC, but the LSDV model, which includes company dummies, yields more highly significant coefficients for the yield level and smaller standard errors of the estimate.

Basic OLS model

The multiple regression results for each of the six dependent variables are found in Table 4-4.[4] Ordinary least squares estimators have two interesting properties in the models specified. First, if a particular set of assets is broken into mutually exclusive subsets and equations with the same explanatory variables are estimated for all of the subsets, the sum of the regression coefficients across subsets will equal the respective coefficients of the equation for the initial set. Also, if total assets are broken into mutually exclusive subsets and identical equations are estimated for all subsets, the sum of the constants across the equations will be unity and the sum of the coefficients of each independent variable across the equations will be zero.[5] Two examples of the first identity are found in Table 4-4. Since $\frac{B}{TA} + \frac{M}{TA} = \frac{F}{TA}$ for each observation, the sum of the regression coefficients for $\frac{B}{TA}$ and $\frac{M}{TA}$ for each independent variable equals exactly the regression coefficient for the corresponding independent variable for $\frac{F}{TA}$. The same statement for $\frac{CS}{TA}$, $\frac{PS}{TA}$, and $\frac{TS}{TA}$ holds only approximately because, in some cases, savings and loan shares were included in $\frac{TS}{TA}$ but not in $\frac{CS}{TA}$

---

[4]The same regressions for 104 U.S. and Canadian companies are in Appendix IV-4. The results of the 104 company sample are virtually identical to those of the 92 company sample discussed here.

[5]The second identity is established by William C. Brainard and James Tobin in "Pitfalls in Financial Model Building," American Economic Review, LVIII (May, 1968),99-122. The first identity necessarily follows from the second one.

TABLE 4-4

CROSS-SECTIONAL/TIME-SERIES RESULTS FOR 92 COMPANIES, 1957-71, USING YIELD LEVEL PROXY

	Dependent Variables					
Independent Variables	$\frac{B}{TA}$	$\frac{M}{TA}$	$\frac{F}{TA}$	$\frac{CS}{TA}$	$\frac{PS}{TA}$	$\frac{PS}{TA}$
CONSTANT	.472438	.512555	.984993	-.037508	-.015625	-.051986
$\frac{R}{TA}$	-.236914 (-5.43)*	-.542547 (-11.53)	-.779461 (-22.60)	.502262 (32.30)	.173524 (19.65)	.675536 (37.27)
YC	-.012237 (-7.97)	-.007712 (-4.65)	-.019948 (-16.43)	.003297 (6.02)	.002091 (6.72)	.005341 (8.37)
SD	.001833 (.34)	.034059 (5.81)	.035892 (8.36)	-.013156 (-6.80)	-.002386 (-2.17)	-.015996 (-7.09)
NL	.050153 (10.09)	-.057770 (-10.76)	-.007617 (-1.94)	.011853 (6.69)	.005940 (5.90)	.017077 (8.26)
NC	.005056 (.65)	.016279 (1.95)	.021335 (3.48)	-.015775 (-5.71)	.000720 (.46)	-.015150 (-4.70)
Std. Error of Estimate	.0785	.0848	.0621	.0280	.0159	.0326
R^2	.1799	.1613	.3838	.4872	.2756	.5594
$\bar{R}^2$	.1769	.1582	.3816	.4854	.2829	.5578
F-Ratio	60.29	52.83	171.19	261.11	104.53	348.95

*t-statistics are given in parentheses beneath their respective coefficients. $\bar{R}^2$ is the coefficient of determination corrected for loss of degrees of freedom.

or $\frac{PS}{TA}$. The second identity does not hold exactly since bonds, mortgages, and equities comprise only 86 percent of total assets, but, nevertheless, the sum of the constants is in the neighborhood of unity and the sum of the coefficients of each independent variable is in the neighborhood of zero.

Table 4-4 indicates the coefficients of the surplus and yield level variables are highly significant and have the signs predicted by the chance-constrained model in every case. The most important independent variable is $\frac{R}{TA}$, which has highly significant t-statistics shown in parentheses. Over the time period covered in this study and for the companies in this sample, the amount of equity relative to total assets appears to be the most important single determinant of the structure chosen by large U. S. life insurance companies. The coefficients of YC also are highly significant for all six equations. These results indicate that life companies with higher risk-bearing capacities (higher surplus) tend to invest more heavily in riskier, higher-return equities and less in fixed income securities. In addition, a higher level of financial yields, given the amounts of surplus and the firms' liabilities to their policyholders, provides the companies with a similar increase in risk-bearing capacity and a significant shift towards riskier types of assets occurs. These two hypotheses will be tested further in subsequent models.

Since the three dummy variables in Table 4-4 behaved in a consistent manner in this model and other models presented below, the discussion of their role here will apply to these later models, also. The coefficients of SD contain information about the comparative behavior of mutual and stock companies. Given their amount of equity and whether they are chartered or licensed in New York, stock companies tend to hold less equities (and more fixed income securities) than mutual companies. More conservative portfolios were expected of stock companies in the chance-constrained model because their premiums are lower and their assumed rates of return on reserves are higher than mutual firms. Given that the other parameters are the same, stock companies must invest in a safer, lower return portfolio to keep the same probability of solvency. The coefficients of SD could also be explained by assuming that stock company managements are more risk averse, but no economic basis for such an assumption is advanced here. The observed economic differences between the life insurance products sold by stock and mutual companies are consistent with these regression results.

In Table 4-4, the effect of New York regulations should appear in the coefficients of NL for those firms licensed in New York and NC for those firms chartered in New York. Given the other independent variables, companies with New York licenses hold significantly more bonds, less mortgages, and more common and preferred stock than the unlicensed companies. The coefficients of NC would indicate differences between the New York chartered companies and companies chartered elsewhere but licensed to sell in New York.

The New York chartered companies appear to hold more mortgages and less common stock. On the other hand, New York regulations have not been binding on the demand for preferred stock. Given their surplus, companies chartered in New York tend to hold roughly 0.4 percent (.011853-.015775) less of their total assets in common stock than companies not subject to New York regulations. Given their surplus, companies licensed and chartered in New York also tend to hold approximately 5 and 3 percent, respectively, less of their total assets in mortgages than companies without New York licenses.

Since these regression results indicate that New York regulations on common stock are not binding, some further analysis is warranted. Over the 1957-71 time period during which the proportion of assets invested in common stock increased, limits on common stock holdings in New York were raised from the lesser of (a) 5 percent of assets or (b) one-half of surplus to exactly double this (10 percent of assets or 100 percent of surplus) in 1969. Consequently, a prior year such as 1967 is an interesting year to examine the slack available to life insurance companies in this investment category. Table 4-5 shows how heavily the 92 U.S. companies invested in common stock compared to their individual legal limits (of course, 53 of the companies are not subject to these limits). It is obvious that considerable slack existed for most companies.

Since the New York regulations are stated in terms of the acquisition costs of the common stock and not in terms

TABLE 4-5

MARKET VALUE OF INVESTMENTS IN COMMON STOCK AS A FRACTION OF THE NEW YORK LEGAL LIMIT, 1967

$\frac{\text{Market value CS}}{\text{Legal Limit}}$	Companies Not Licensed in NY	Companies Licensed in NY, Chartered Elsewhere	Companies Chartered in NY	Totals
1.0 and above	19	13	4	36
.8 - 1.0	6	4	0	10
.6 - .8	6	5	0	11
.4 - .6	6	1	0	7
.2 - .4	6	4	2	12
0 - .2	10	3	3	16
Totals	53	30	9	92

of market value (the valuation procedure used by the National Association of Insurance Commissioners), Table 4-5 understates the amount of slack available to companies owning common stock which has appreciated in market value after its initial purchase. Since the cost of their stock holdings was not available, it was estimated for each firm by assuming that the cost and market values coincided in 1957 and then cumulating estimated annual net purchases from 1957 forward. The estimated net purchase for a year is the year-end market value less the previous year's market value adjusted for this year's change in Standard Poor's 500 Stock Index. Of course, this estimated cost figure used in Table 4-6 is only an attempt to deflate the investment in common stock for the generally rising prices over the period of this study and would be only an approximation of the actual historical cost for each company. These figures indicate a significant reluctance on the part of the majority of life insurance companies to press the New York legal maximum.

New York regulations do not appear to be binding on many firms: most companies appear to maintain a slack exceeding 40 percent of their legal maximum constraints. While some companies may wish to maintain some slack in order to appease examiners or, more likely, to maintain the capacity to invest more heavily when investment opportunities are better, the New York limits simply exceed what many managements want to invest. Of course, the restriction of the lessor of 3 percent of assets or 33 percent of surplus, which existed

TABLE 4-6

ESTIMATED COST OF INVESTMENTS IN COMMON STOCK AS A FRACTION OF THE NEW YORK LEGAL LIMIT, 1967

Estimated cost CS / Legal Limit	Companies Not Licensed in NY	Companies Licensed in Ny, Chartered Elsewhere	Companies Chartered in NY	Totals
1.0 and above	8	5	0	13
.8 - 1.0	1	1	2	4
.6 - .8	4	3	1	8
.4 - .6	14	9	0	23
.2 - .4	8	2	1	11
0 - .2	18	10	5	33
Totals	53	30	9	92

prior to 1957, would be exceeded by most companies, and the 5 percent of assets/50 percent of surplus limit, had it not been relaxed in 1969, may have proved binding to more firms in later years. While New York regulations as a whole, both on the general qualitative and quantitative composition of the portfolio and on the marketing and characteristics of the life insurance product, may have a significant impact, clearly the restriction on common stock is not universally binding.

It is obvious from the interpretation of the regression coefficients of the six equations in Table 4-4 that life companies behave roughly as the chance-constrained model predicts they should. However, the explanatory power of the six equations varies considerably and leaves much portfolio behavior among the companies unexplained.

The equations for $\frac{B}{TA}$ and $\frac{M}{TA}$ have the lowest coefficients of determination; this was expected since bonds and mortgages tend to be close substitutes. The equation for their sum, $\frac{F}{TA}$, has a higher $\bar{R}^2$ and a lower standard error of the estimate than either $\frac{B}{TA}$ or $\frac{M}{TA}$. The equations for $\frac{CS}{TA}$ are better than for $\frac{PS}{TA}$, a situation which is understandable since the legal and economic status of common stock is less ambiguous than that of preferred stock. The equations for $\frac{CS}{TA}$ and $\frac{TS}{TA}$ are the strongest of the six. However, because of considerable between firm variation which is not explained by the independent variables, the next section uses a model which eliminates these interfirm differences.

Basic LSDV model

Because the residuals of the OLS model above had non-zero means for each company in the cross section, a least-squares dummy-variable (LSDV) model can be utilized to improve the efficiency of the estimates. If, as appears to be the case above, the OLS model has the structure

$$Y_{ij} = a + bX_{ij} + u_{ij} \qquad \begin{array}{l} i = 1, n \text{ firms} \\ j = 1, t \text{ periods} \end{array}$$

in which the residuals are

$$u_{ij} = v_i + w_{ij}$$

(v and w with zero means and constant variances), the error associated with each firm, v_i, can be removed by using company dummies

$$Y_{ij} = \alpha_i + bX_{ij} + w_{ij}$$

leaving only the purely random error, w_{ij}. The LSDV technique involves a loss of degrees of freedom and eliminates the between firm variation. Ignoring potential sources of between firm variation is justifiable here on two grounds. First, many sources of between firm variation are not modeled here because of their complexity or because of the inability to ascribe numerical values to them. Second, while some cross-sectional variables such as $\frac{R}{TA}$, SD, NL, and NC are highly significant, the LSDV technique helps focus on the within firm variation due to covariates such as $\frac{R}{TA}$ and YC.

The effect of adding company dummies as independent variables can be found by comparing the equations from estimating the demand for the six dependent variables using the

LSDV model in Table 4-7 with the similar equations of Table 4-4, which was the OLS model. The dummies NL, NC, and SD of Table 4-4 are dropped because they would form a singularity with the company dummies. Table 4-8 provides a comparison of the summary statistics for these equations: the addition of company dummies substantially reduces the standard errors; the source of most of the unexplained variance in Table 4-4 is inter-company variation.

For the equations explaining $\frac{B}{TA}$, $\frac{M}{TA}$, and $\frac{F}{TA}$, the absolute sizes of the $\frac{R}{TA}$ and their corresponding t-values were reduced. The coefficients for YC were practically unchanged and their corresponding t-values substantially increased. For the equations explaining $\frac{CS}{TA}$, $\frac{PS}{TA}$, and $\frac{TS}{TA}$, the absolute sizes of the coefficients of $\frac{R}{TA}$ are substantially the same and the t-values are reduced, though still highly significant. The absolute sizes of the coefficients of YC are about the same and the t-values are larger and more significant.

The use of the LSDV model improved the characteristics of the equations estimating the six dependent variables and did not alter the role of the relative amount of surplus or the yield level proxy for these variables. The coefficients of the company dummies, which are not reported in Table 4-7, exhibited statistically significant differences from each other. Analysis of covariance tests can be used

TABLE 4-7

CROSS-SECTIONAL/TIME-SERIES RESULTS USING LSDV MODEL
COEFFICIENTS OF COMPANY DUMMIES ARE NOT REPORTED

Independent Variables	Dependent Variable $\frac{B}{TA}$	$\frac{M}{TA}$	$\frac{F}{TA}$	$\frac{CS}{TA}$	$\frac{PS}{TA}$	$\frac{TS}{TA}$
$\frac{R}{TA}$	-.092088 (-1.68)	-.188664 (-3.67)	-.280751 (-6.67)	.502594 (27.17)	.156236 (11.06)	.654214 (25.65)
YC	-.012370 (-15.95)	-.008037 (-11.05)	-.020408 (-34.33)	.003297 (12.61)	.002107 (10.56)	.005361 (14.88)
Std. Error of Estimate	.0396	.0371	.0304	.0133	.0102	.0184
R^2	.8046	.8494	.8623	.8910	.7218	.8690
$\bar{R}^2$	.7905	.8385	.8524	.8831	.7016	.8595
F-Ratio	56.95	78.00	86.62	113.03	35.87	91.70

TABLE 4-8

SUMMARY STATISTICS FROM OLS AND LSDV MODELS

Dependent Variable	Dummies	Standard Error	R^2
$\frac{B}{TA}$	SD, NL, NC	.0785	.1799
	Co. Dummies	.0396	.8046
$\frac{M}{TA}$	SD, NL, NC	.0848	.1613
	Co. Dummies	.0371	.8494
$\frac{F}{TA}$	SD, NL, NC	.0621	.3838
	Co. Dummies	.0304	.8623
$\frac{CS}{TA}$	SD, NL, NC	.0280	.4872
	Co. Dummies	.0133	.8910
$\frac{PS}{TA}$	SD, NL, NC	.0159	.2756
	Co. Dummies	.0102	.7218
$\frac{TS}{TA}$	SD, NL, NC	.0326	.5594
	Co. Dummies	.0184	.8690

to show the significance of interfirm variation.[6] Covariance analysis is used here to test differences in firm intercepts (assuming common slopes), to test differences in firm slopes (given different intercepts), and to test overall homogeneity of the equations among the firms. The residual sums of squares for each dependent variable for three equations using $\frac{R}{TA}$ and YC as independent variables are shown in Table 4-9. The addition of company dummies (different intercepts for each firm) to a simple equation with a common intercept and common slopes for all firms leads to a highly significant reduction in the residual sum of squares, as established by the value for F_1: the company dummies are not homogeneous. F_2 indicates that the slopes for $\frac{R}{TA}$ and YC are not homogeneous across firms, given different intercepts. With non-homogeneous slopes and intercepts, the overall coefficients (for dummies and slopes together) prove non-homogeneous as well. While the hypotheses of the chance-constrained model are tested appropriately using common slope coefficients for the sample of life insurance companies, statistically significant individual differences in behavior exist.

An alternative to the OLS and LSDV models presented above is the generalized least squares (GLS) method of pooling cross-sectional/time-series data.[7] The GLS estimator

---

[6]For an explanation of the covariance analysis used here, see J. Johnston, Econometric Methods (2nd ed.; New York: McGraw Hill, 1972), pp. 192-99.

[7]G.S. Maddala, "The Use of Variance Components Models in Pooling Cross Section and Time Series Data," Econometrica, XXXXIX (March, 1971), 341-58.

TABLE 4-9

ANALYSIS OF COVARIANCE TESTS OF HOMOGENEITY ACROSS FIRMS

Source	Sum of Squares	Degrees of Freedom	Dependent Variables					
			$\frac{B}{TA}$	$\frac{M}{TA}$	$\frac{F}{TA}$	$\frac{CS}{TA}$	$\frac{PS}{TA}$	$\frac{TS}{TA}$
Firm slopes and firm intercepts	Residual	1104	.66539	.68553	.31362	.08691	.03534	.12222
	Incremental	182	1.35331	1.08816	.87174	.14225	.09826	.31336
Common slopes and firm intercepts	Residual	1286	2.01870	1.77369	1.18536	.22916	.13360	.43558
	Incremental	91	7.23360	9.54164	4.46771	.94872	.22888	1.19695
Common slopes and intercept	Residual	1377	9.25230	11.31533	5.65307	1.17788	.36248	1.63253
F_1: Test of differential intercepts		91,1286	55.20	77.76	56.57	64.20	25.12	41.14
F_2: Test of differential slopes		182,1104	12.34	9.63	16.86	9.93	16.87	15.55
F_3: Test of overall homogeneity		273,1104	52.19	62.71	68.85	50.76	37.43	49.97

would have less variance than either the OLS or the LSDV estimators used here and thus be more efficient. In this case, the estimates would be very close to the LSDV model because of the large size of the between group variation compared to the within group variation.[8] The efficiency possible with the GLS estimator does not differ appreciably from the LSDV estimator in this sample.

Overall, the least-squares dummy-variable model yields highly significant results which are consistent with the chance-constrained portfolio model. The significance of the yield level variable was enhanced by the use of company dummies. The magnitude of the between firm variation implies that unexplained firm-related phenomena are of great importance, however the amount of surplus and the yield level remain highly significant factors in explaining portfolio structure. The next section expands the results of the basic OLS and LSDV models and a subsequent section deals with the positive serial correlation observed in these models.

Extensions of the Basic Models

Despite their inherent limitations, it is possible to extend the basic models to consider the effect of relative yields and of company size on portfolio choices and to compare the U. S. and Canadian companies.

---

[8] Ibid., 343.

<u>The effects of relative yields.</u>

In the chance-constrained model, the analytical role of relative yields on different securities is complex, which makes the empirical significance of yields particularly interesting. For two securities that are substitutes in the model, if the yield on one security increases relative to the other, <u>ceteris paribus</u>, management should invest more in the security with the increased relative yield (and less in the other). Also, as established in the previous chapter, the demand for a security can be related inversely to its own yield. The function of relative yields may be found by substituting the yield ratios $\frac{YM}{YC}$ or $\frac{YP}{YC}$ into the OLS model in place of the yield level variable YC. The dependent variables considered here in Table 4-10 are the relative amounts of bonds, mortgages, and preferred stock: common stock is not used as a dependent variable because no satisfactory <u>ex ante</u> measure of the expected yield on common stock exists.[9]

A comparison of the equations for $\frac{B}{TA}$, $\frac{M}{TA}$, and $\frac{PS}{TA}$ in Tables 4-10 and 4-4 (which used the yield level) shows that the coefficients and the t-values of the relative amount of surplus, the stock dummy, the New York license dummy, and the New York charter dummy are quite similar. Consequently,

---

[9]Appendix IV-5 contains equations similar to Table 4-10 using yield differences instead of yield ratios. The two forms of the relative yield variables appear interchangeable. The next major section of this chapter includes a model using yield ratios in the LSDV model with similar results.

TABLE 4-10

CROSS-SECTIONAL/TIME-SERIES RESULTS FOR 92 COMPANIES, 1957-71, USING YIELD RATIOS

Independent Variables	Dependent Variable					
	$\frac{B}{TA}$	$\frac{M}{TA}$	$\frac{B}{TA}$	$\frac{M}{TA}$	$\frac{PS}{TA}$	$\frac{PS}{TA}$
CONSTANT	.2344	.3921	.1709	.4194	.03608	.03355
$\frac{YM}{YC}$	.1434 (8.61)	.06572 (3.63)	.05169 (1.43)	.1051 (2.68)		-.008721 (-1.19)
$\frac{YP}{YC}$			.1798 (2.87.)	-.07724 (-1.13)	-.04189 (-7.12)	-.02842 (-2.23)
$\frac{R}{TA}$	-.2267 (-5.21)	-.5394 (-11.41)	-.2158 (4.96)	-.5441 (-11.47)	.16966 (19.98)	.1702 (19.22)
SD	.001188 (.22)	.03386 (5.76)	.0004982 (.09)	.03416 (5.81)	-.00214 (-1.95)	-.002177 (-1.98)
NL	.05028 (10.15)	-.05773 (-10.72)	.05141 (10.20)	-.05779 (-10.73)	.00589 (5.86)	.005900 (5.87)
NC	.005157 (.67)	.01631 (1.94)	.005265 (.68)	.01626 (1.94)	.00068 (.44)	.006876 (.44)
Std. Error	.0782	.0851	.9782	.0850	.0159	.0159
R^2	.1859	.1561	.1908	.1569	.2784	.2791
$\bar{R}^2$	.1830	.1531	.1872	.1532	.2758	.2760
F-Ratio	62.76	50.84	53.94	42.59	106.01	88160
Degrees of Freedom	(5,1374)	(5,1374)	(6,1373)	(6,1373)	(5,1374)	(6,1373)

the comments made for Table 4-4 about these independent variables apply equally well to these equations using relative yields instead of a proxy yield level and only the yield variables merit discussion.

In Table 4-10, the equations for $\frac{B}{TA}$ and $\frac{PS}{TA}$ have slightly lower standard errors of the estimate and slightly higher coefficients of determination than the corresponding equations in Table 4-4 and the reverse is true of the equations for $\frac{M}{TA}$. The use of relative yields instead of a proxy for the yield level of all financial assets did not significantly improve the explanatory power of the regression equations. For securities which are substitutable, portfolio shifts toward those securities with relatively higher yields should occur. No such consistent pattern emerged in the statistical analysis.

In the first two equations of Table 4-10, $\frac{B}{TA}$ and $\frac{M}{TA}$ both appear positively related to the ratio of the mortgage yield to the corporate bond yield. If bonds and mortgages are close substitutes, opposite signs would be expected, probably (though not necessarily) a negative sign for $\frac{YM}{YC}$ in the $\frac{B}{TA}$ equation and a positive sign in the $\frac{M}{TA}$ equation. The same sign implies that a decrease in $\frac{YM}{YC}$ would result in life companies purchasing more of other kinds of assets. Adding $\frac{YP}{YC}$ to the $\frac{B}{TA}$ equation implies that if the yield on corporate bonds goes down relative to mortgages and preferred stock, life companies buy more bonds. According to the equations for $\frac{PS}{TA}$, an increase in the yield on preferred stock relative to corporate bonds would induce life companies to buy less preferred stock. These results are not consistent with the predicted signs for mortgages and preferred stock.

These poor results of relative yields may be caused by basic statistical and economic problems. As seen in Table 4-11, the yield variables are highly intercorrelated.

TABLE 4-11

CORRELATIONS AMONG YIELD VARIABLES, 1957-71

	YC	YM	YP	$\frac{YM}{YC}$	$\frac{YP}{YC}$
YC	1.000				
YM	.988	1.000			
YP	.986	.980	1.000		
$\frac{YM}{YC}$	-.962	-.915	-.932	1.000	
$\frac{YP}{YC}$	-.848	-.819	-.752	.887	1.000

With such highly correlated independent variables and only 15 annual observations per company, multicollinearity is a problem when more than one yield variable is used in an equation. In addition, there are secular trends in yield levels, differentials, and ratios as well as in the dependent variables. The ordinary least squares model used above might find a spurious relationship which is statistically significant because of the small number of annual observations.

An alternative explanation of the poor performance of relative yields may be that capital markets price securities quite rationally, given their maturity, liquidity, and risk of default, such that a change in relative yields may reflect these factors and not imply that one security is relatively more desirable, all things considered. In any case, relative yields prove troublesome here and they are reconsidered in subsequent models.

The effect of company size

The previous results indicate that most of the between company variation remains unexplained. While no explicit hypotheses about company size were derived from the chance-constrained model of Chapter III, the size of firms could be a source of cross-sectional variation. Increasing returns to scale may occur when increased size helps overcome indivisibilities and permits increased specialization of resources and personnel. If there are fixed costs associated with each market in which life companies purchase securities, a small company may forego some types of securities to keep its operating fixed costs low. For example, a small company may not be able to afford both mortgage and bond specialists on its staff. Larger companies are able to hire more specialized talent and effectively cover all investment areas. Other cost differences based on company size exist. Large companies may have lower brokerage costs because they make larger transactions. Large companies may use their own departments and rely less heavily on mortgage brokers and mortgage banks and large companies may have advantages over smaller ones when competing for privately placed securities. A smaller company has a tradeoff between operating costs and the risk reduction accompanying greater diversification and, faced with this choice, a small company may choose to invest in a narrower range of capital market instruments.

In Table 4-12, the log of total assets is used as an independent variable to estimate the impact of company size on portfolio choices.[10] The other variables ($\frac{R}{TA}$, YC, SD, NL, and NC) are identical to the regressions reported in Table 4-4. While the additional size variable does reduce the standard errors of the estimate slightly, it is apparent that the substantial inter-company variation noted previously still remains. The coefficients and their t-values are basically the same in Table 4-4 and Table 4-12 except that in the latter the absolute sizes of the NL coefficients are reduced and the NC coefficient in the $\frac{M}{TA}$ equation becomes more significant when ln(TA) is included as an independent variable. Since companies licensed or chartered in New York tend to be the larger companies, the coefficients of NL and NC were affected slightly.

Since the other coefficients have been discussed in a previous section, ln(TA) is the only variable requiring analysis here. If the fraction of total assets invested in a security is positively related to the size of the company,

---

[10] Other size variables used were the ordinal rank of a company's size, total assets in millions of dollars, and the reciprocal of total assets. The log of total assets used here resulted in the lowest standard errors,but the results of all size variables were quite similar.

TABLE 4-12

CROSS-SECTIONAL/TIME-SERIES RESULTS FOR 92 COMPANIES, 1957-71, USING LN(TA)

	Dependent Variable					
Independent Variables	$\frac{B}{TA}$	$\frac{M}{TA}$	$\frac{F}{TA}$	$\frac{CS}{TA}$	$\frac{PS}{TA}$	$\frac{PS}{TA}$
CONSTANT	.418746	.594372	1.013118	-.066406	-.029013	-.093536
$\frac{R}{TA}$	-.233967 (-5.42)	-.547038 (-11.85)	-.781005 (-22.73)	.503848 (33.14)	.174259 (20102)	.677817 (38.73)
SD	.005507 (1.02)	.028461 (4.91)	.033968 (7.88)	-.011178 (-5.86)	-.001470 (-1.35)	-.013153 (-5.99)
YC	-.014018 (-9.00)	-.004998 (-3.00)	-.019105 (-15.34)	.002339 (4.27)	.001647 (5.25)	.003963 (6.28)
NL	.040556 (7.72)	-.043046 (-7.69)	-.002590 (-.62)	.006688 (3.62)	.003547 (3.35)	.009650 (4.53)
NC	-.000538 (-.07)	.024803 (3.00)	.024265 (3.94)	.018785 (-6.89)	-.000675 (-.43)	-.019479 (-6.20)
ln(TA	.010458 (5.26)	.015935 (-7.50)	-.005478 (-3.46)	.005268 (8.04)	.--2608 (6.51)	.008093 (10.04)
Std. Error	.0780	.0831	.0619	.0274	.0157	.0315
F-Ratio	55.83	55.16	145.79	238.43	96.78	328.72
$\bar{R}^2$	.1926	.1907	.3865	.5081	.2942	.5878

that security can be labeled a superior asset; if the same relationship is negative, the security can be labeled an inferior asset. Using these definitions, the coefficients of ln(TA) imply that bonds, common stock, preferred stock, and, of course, common and preferred stock combined are superior assets, and that mortgages and bonds and mortgages combined are inferior assets. The heavier investment in equities and lower investment in fixed income securities by large companies may result because large companies have the opportunity to diversify more extensively and invest slightly more in riskier types of securities and still maintain the same probability of solvency as a smaller firm. Since the primary and secondary markets for martgages are not as well developed as for bonds, the coefficients of ln(TA) in the $\frac{B}{TA}$ and $\frac{M}{TA}$ equations are interesting. Since the overhead and variable expenses of a mortgage department are large, it is a natural presumption that large companies might enjoy significant economies of scale in this type of investment and invest more in mortgages than smaller companies. The opposite occurred here. It is possible that the companies in this study were of sufficient size that they are generally able to invest efficiently in mortgages, given the institutional characteristics of that market. The inferiority of mortgages can result from spatial preferences for mortgages; increasing distances may entail more expense and risk. In addition, if the supply of mortgages to a life company is more yield-elastic than the supply of bonds, some life insurance companies may have better yields available to them in mortgages due to their location, bond yields being more homogeneous nationwide. Geography must explain some of the preferences for bonds and mortgages.

Since the sizes of the insurance companies were increasing over the period of this study, it is necessary to establish whether the conclusions about the significance of company size were dependent on time. In Table 4-13, company dummies are used to moderate for the effects of cross-sectional variation and year dummies are used to account for possible time-related variation. Yields are not used as a variable since they are assumed homogeneous at a point in time. Since $\frac{R}{TA}$ covaries with time and firm, its significance may also be ascribed to cross-sectional or time-related variation. In Table 4-13, the coefficients and t-values for $\frac{R}{TA}$ and ln(TA) are very similar for the simple least squares equations and the equations with year dummies, so the results obtained previously are obviously not due to a spurious time trend. When company dummies are used, most of the coefficients are markedly different, reflecting the time-series trends.
It is obvious from Table 4-13 that the coefficients estimated using the pooled data are basically due to cross-sectional variation, since in the regressions for $\frac{B}{TA}$, $\frac{M}{TA}$, and $\frac{F}{TA}$, the coefficients found using company dummies were significantly different from the regressions with year dummies and a constant with no dummies. Hypotheses about the comparative behavior of life companies due to different surplus levels

TABLE 4-13

ESTIMATES USING COMPANY AND TIME DUMMIES

	Independent Variables					
Dependent Variable	$\frac{R}{TA}$	ln(TA)	Dummies	Degrees of Freedom	Std. Error of Estimate F-Ratio	$\bar{R}^2$
$\frac{B}{TA}$	-.3087 (-8.14)	.0135 (7.46	constant	(2,1377)	.0820 79.54	.1023
$\frac{B}{TA}$	.1177 (2.05)	-.0604 (15.88)	company	(93,1286)	.0396 56.84	.7902
$\frac{B}{TA}$	-.2528 (-6.86)	.0185 (10.29)	year	(16,1363)	.0787 19.11	.1736
$\frac{M}{TA}$	-.3663 (-9.19)	-.0248 (-13.02)	constant	(2,1377)	.0862 104.66	.1307
$\frac{M}{TA}$	-.1018 (-1.85)	-.0281 (-7.69)	company	(93,1286)	.0380 73.89	.8310
$\frac{M}{TA}$	-.3871 (-9.68)	-.0252 (-12.93)	year	(16,1363)	.0853 15.89	.1473
$\frac{F}{TA}$	-.6750 (-21.32)	-.0113 (-7.45)	constant	(2,1377)	.0684 230.39	.2496
$\frac{F}{TA}$	.0159 (.33)	-.0885 (-27.72)	company	(93,1286)	.0333 69.89	.8229
$\frac{F}{TA}$	-.6399 (-21.50)	-.0067 (-4.63)	year	(16,1363)	.0635 48.27	.3542

TABLES 4-13--Continued

	Independent Variables					
Dependent Variable	$\frac{R}{TA}$	ln(TA)	Dummies	Degrees of Freedom	Std. Error of Estimate F-Ratio	$\bar{R}^2$
$\frac{CS}{TA}$	.4594 (34.99)	.0069 (11.05)	constant	(2,1377)	.0284 616.33	.4716
$\frac{CS}{TA}$	.4423 (23.08)	.0171 (13.44)	company	(93,1286)	.0133 114.91	.8848
$\frac{CS}{TA}$	.4539 (34.30	.0063 (9.83)	year	(16,1363)	.0282 79.68	.4772
$\frac{PS}{TA}$	.1642 (22.35)	.0036 (10.34)	constant	(2,1377)	.0159 263.53	.2758
$\frac{PS}{TA}$	.1104 (7.65)	.0126 (13.14)	company	(93,1286)	.0100 38.05	.7142
$\frac{PS}{TA}$	.1598 (21.60)	.0032 (8.83)	year	(16,1363)	.0158 35.26	.2843
$\frac{TS}{TA}$	.6222 (40.88)	.0104 (14.24)	constant	(2,1377)	.0329 846.93	.5509
$\frac{TS}{TA}$	.5492 (21.09)	.0294 (17.00)	company	(93,1286)	.0180 96.43	.8655
$\frac{TS}{TA}$	.6123 (40.17)	.0093 (12.55)	year	(16,1363)	.0325 111.23	.5612

and sizes were tested adequately with the pooled data: time-related trends did not bias the acceptance of these hypotheses.

In this section, it appears that fixed income securities are an inferior good, equities are a superior good, and, interestingly, that bonds proved superior to mortgages. Yield variables, which are not analyzed with cross-sectional data, are examined in more depth in subsequent parts of this chapter.

Comparison of U.S. and Canadian companies

The 12 Canadian companies in the sample did invest in a statistically significant different manner than their 92 U. S. counterparts, as is shown by the Canadian dummy (CD = 1 if Canadian, CD = 0 if U. S.) of Table 4-14. The percentage of total assets invested in bonds was 0.8 percent more for Canadian firms, 2.7 percent less for mortgages, 2.3 percent more for common stock, and 0.3 percent more for preferred stock, given the other independent variables. The differences for mortgages, common stock, and preferred stock are significant at the .01 level.

Further statistical evidence of the differences is obtained by estimating separate equations for U. S. firms, Canadian firms, and both sets combined and performing a Chow test on the hypothesis that the structures of the equations are homogeneous.[11] Using the structure of the basic

---

[11] Gregory C. Chow, "Tests of Equality Between Sets of Coefficients in Two Linear Regressions," Econometrica, XXVIII (July, 1960), 591-605.

TABLE 4-14

BASIC OLS MODEL: CROSS-SECTIONAL/TIME-SERIES RESULTS WITH CANADIAN DUMMY

	Dependent Variables					
Independent Variables	$\frac{B}{TA}$	$\frac{M}{TA}$	$\frac{F}{TA}$	$\frac{CS}{TA}$	$\frac{PS}{TA}$	$\frac{TS}{TA}$
CONSTANT	.486931	.501647	.988577	-.038833	-.013135	-.050469
$\frac{R}{TA}$	-.192.75 (-4.51)	-.585276 (-12.56)	-.777451 (-24.09)	.503637 (34.29)	.173711 (20.55)	.674523 (39.59)
YC	-.014412 (-9.92)	-.005969 (-3.76)	-.020381 (-18.52)	.003605 (7.20)	.001732 (6.01)	.005233 (9.01)
SD	-.088652 (-1.72)	.043219 (7.87)	.034567 (9.09)	-.013503 (-7.81)	-.002917 (-2.93)	-.016426 (-8.19)
NL	.045.98 (9.39)	-.054643 (-10.39)	-.009445 (-2.59)	.010965 (6.61)	.005312 (5.57)	.015582 (8.10)
NC	.006094 (.79)	.015988 (1.89)	.022082 (3.76)	-.015283 (-5.72)	.000927 (.60)	-.014401 (-4.65)
CD	.008271 (1.25)	-.026567 (-3.67)	-.018296 (-3.65)	.023345 (10.23)	.003494 (2.66)	.027334 (10.32)
Std. Error of Estimate	.0790	.0864	.0598	.0272	.0157	.0316
R^2	.1771	.1605	.3946	.4808	.2593	.5510
$\bar{R}^2$	.1739	.1572	.3923	.4788	.2564	.5493
F-Ratio	55.69	49.48	168.72	239.68	90.59	317.68

OLS equation in Table 4-4, the F tests that the two sets are identical with (6,1548) degrees of freedom are as follows for each dependent variable:

$$\frac{B}{TA}: \quad F = 11.50$$

$$\frac{M}{TA}: \quad F = 8.72$$

$$\frac{F}{TA}: \quad F = 3.02$$

$$\frac{CS}{TA}: \quad F = 18.75$$

$$\frac{PS}{TA}: \quad F = 4.33$$

$$\frac{TS}{TA}: \quad F = 19.10$$

While the portfolios are significantly different using the t-tests of the Canadian dummy or the Chow test, in absolute terms the Canadian companies are remarkably similar, with the largest absolute difference in an investment category being less than 3 percent.[12] Their larger investment in common stock is the largest relative difference but, if the strong upward trend in $\frac{CS}{TA}$ for U.S. companies continues, the existing Canadian levels should be reached by most U.S. companies in a few years.

---

[12]The results of estimating differential slopes and intercepts for Canadian companies are given in Appendix IV-6. In these equations, multicollinearity reduced the significance of the results.

Cross-Sectional/Time-Series Results Using Adjustments for First-Order Autocorrelation and Interfirm Variance

In this section, an adjustment for first-order autocorrelation is integrated into the LSDV model to estimate the demand for all six dependent variables using surplus and the yield level as independent variables and to estimate the demand for bonds and mortgages with surplus and relative yields. When serial correlation exists, OLS estimates of the coefficients are unbiased, but the sampling variances of the coefficients and the standard errors of the estimate are underestimated and, in addition, the OLS technique is relatively inefficient.[13] Since only 15 annual observations are available for each company, the significance of the previous results and the relative efficiency of the estimates are important considerations. If we assume a relationship of the form

$$Y_t = X_tB + u_t$$

which has the first-order autoregressive scheme

$$u_t = \rho u_{t-1} + v_t \qquad |\rho| \leq 1$$

where ρ is known *a priori*, the effect of autocorrelation is easily removed as follows:

$$Y_t = X_tB + \rho u_{t-1} + v_t$$

$$Y_{t-1} = X_{t-1}B + u_{t-1}$$

$$\rho Y_{t-1} = \rho X_{t-1}B + \rho u_{t-1}$$

---

[13] Johnston, *Econometric Methods*, pp. 246-49.

$$(Y_t - \rho Y_{t-1}) = (X_t - \rho X_{t-1})B + v_t$$

$$Y_t^* = X_t^* B + v_t$$

Hildreth and Lu give a useful iterative technique for determining ρ in which the criterion for the selection of ρ is to iterate ρ over the interval (-1,1) to find that ρ which minimizes the sum of squares of error.[14] With the number of degrees of freedom fixed, the minimum sum of squares of error corresponds to the minimum standard error of the estimate. A direct estimate of ρ (such as $\rho = 1 - \frac{D}{2}$)[15] is not possible from the Durbin-Watson statistic since pooled data is utilized.

This first-order autoregressive scheme is combined with the LSDV model in the following equation:

$$Y_{ij}^* = \alpha_i + B_1 X_{1ij}^* + B_2 X_{2ij}^* + v_{ij}$$

Briefly, $Y^* = Y - \rho Y_{t-1}$, α_i = company dummy, $X_1^* = X_{1t} - \rho X_{1(t-1)}$, $X_2^* = X_{2t} - \rho X_{2(t-1)}$, B_1 and B_2 are the coefficients of the adjusted independent variables, and v_{ij} is the error term. Y is the proportion of assets invested in a particular asset, X_1 is the relative amount of surplus, and X_2 is the yield level proxy. No adjustment is made to the dummies since

---

[14] Clifford Hildreth and John Y. Lu, "Demand Relations with Autocorrelated Disturbances," Michigan State University Technical Bulletin 276, (November, 1960), 1-76.

[15] A.S. Goldberger, Econometric Theory (New York: John Wiley and Sons, Inc., 1964), p. 244.

they are constant over the time period and would become zero when $\rho = 1.0$

In Tables 4-15-a through 4-15-f, several iterations of ρ are presented for each of the six dependent variables. In the first column of each table where $\rho = 0.0$, the structure of the equations is the same as the basic LSDV model in Table 4-7, except the 1957 observation is omitted in order to obtain a lagged year's value for each company; this leaves 14 observations for each company and NT = (92)(14) = 1288 observations. The equations with 14 and 15 observations per firm are virtually identical. In the final column of each table where $\rho = 1.0$, the variables are simply first differences (except for the dummies, which are not transformed).

As ρ is increased, in all cases the coefficients and their t-values exhibit clear trends so that the results of using more precise ρ's can be easily guessed through interpolation. For each dependent variable, the unreported company dummies became less significant as ρ was increased and the coefficients of determination generally declined. The minimum standard errors occurred at approximately $\rho = .90$ for bonds, $\rho = .87$ for mortgages, $\rho = .73$ for common stock, and $\rho = .93$ for preferred stock.

In the equations for $\frac{B}{TA}^*$. $\frac{M}{TA}^*$, and $\frac{F}{TA}^*$ in Table 4-15, the coefficients of $\frac{R}{TA}^*$ and their corresponding t-values increase as ρ increased. The coefficients of YC^* have lower significance as ρ is increased, but remain significant at the .01 level. In the equity equations, the coefficients of

TABLE 4-15-a

CROSS-SECTIONAL/TIME-SERIES REGRESSIONS INCORPORATING AN ADJUSTMENT FOR FIRST ORDER AUTOCORRELATION AND COMPANY DUMMIES. COEFFICIENTS OF THE COMPANY DUMMIES ARE NOT REPORTED

	Dependent variables is $\frac{B}{TA} - (r\ \frac{B}{TA})_{lagged}$					
Independent variables	r-0.0	r-0.5	r=0.7	r=0.8	r=0.9	r=1.0
$\frac{R}{TA} - r(\frac{R}{TA})_{lagged}$	-.1227 (-2.19)	-.1729 (-3.45)	-.2017 (-4.46)	-.2093 (-4.87)	-.2130 (-5.12)	-.2209 (-5.35)
$YC - rYC_{lagged}$	-.0119 (-15.48)	-.00980 (-12.87)	-.00778 (-9.62)	-.00636 (-7.33)	-.00490 (-5.13)	-.00411 (-3.90)
Standard Error of the Estimate	.0380	.0222	.0178	.0165	.0160	.0165
R^2	.8131	.7508	.6166	.4506	.2179	.1656
$\bar{R}^2$	.7985	.7314	.5868	.4078	.1570	.1006
F-Ratio (93,1194 d.f.)	55.84	38.69	20.65	10.53	3.58	2.55

TABLE 4-15-b

CROSS-SECTIONAL/TIME-SERIES REGRESSIONS INCORPORATING AN ADJUSTMENT FOR FIRST ORDER AUTOCORRELATION AND COMPANY DUMMIES... COEFFICIENTS OF THE COMPANY DUMMIES ARE NOT REPORTED

	Dependent Variable is $\frac{M}{TA} - (r\frac{M}{TA})_{\text{lagged}}$					
Independent Variables	r=0.0	r=0.5	r=0.7	r=0.8	r=0.9	r=1.0
$\frac{R}{TA} - r(\frac{R}{TA})_{\text{lagged}}$	-.1569 (-3.01)	-.2592 (-5.35)	-.3453 (-7.57)	-.3780 (-8.51)	-.3876 (-8.90)	-.3739 (-8.70)
$YC - rYC_{\text{lagged}}$	-.00797 (-11.18)	-.00865 (-11.76)	-.00836 (-10.25)	-.00752 (-8.40)	-.00567 (-5.68)	-.00276 (-2.51)
Standard Error of the Estimate	.0353	.0214	.0179	.0170	.0167	.0172
R^2	.8624	.8090	.6899	.5376	.2958	.1739
$\bar{R}^2$	.8516	.7941	.6657	.5016	.2409	.1096
F-Ratio (93,1194 d.f.)	80.44	54.37	28.56	14.93	5.39	2.70

TABLE 4-15-c

CROSS-SECTIONAL/TIME-SERIES REGRESSIONS INCORPORATING AN ADJUSTMENT FOR FIRST ORDER AUTOCORRELATION AND COMPANY DUMMIES. COEFFICIENTS OF THE COMPANY DUMMIES ARE NOT REPORTED

	Dependent variable is $\frac{F}{TA} - r(\frac{F}{TA})_{\text{lagged}}$					
Independent Variables	r=0.0	r=0.5	r=0.7	r=0.8	r=0.9	r=1.0
$\frac{R}{TA} - r(\frac{R}{TA})_{\text{lagged}}$	-.2795 (-6.39)	-.4321 (-9.71)	-.5470 (-12.50)	-.5873 (-13.72)	-.6005 (-14.53)	-.5948 (-15.07)
$YC - rYC_{\text{lagged}}$	-.0198 (-33.15)	-.0185 (-27.27)	-.0161 (-20.63)	-.0139 (-16.09)	-.0106 (-11.16)	-.00687 (-6.81)
Standard Error of the Estimate	.0297	.0197	.0172	.0164	.0159	.0158
R^2	.8705	.8007	.6766	.5436	.3534	.2283
$\bar{R}^2$	.8604	.7852	.6514	.5081	.3031	.1682
F-Ratio (93,1194 d.f.)	86.32	51.60	26.86	15.29	7.02	3.80

TABLE 15-d

CROSS-SECTIONAL/TIME-SERIED REGRESSIONS INCORPORATING AN ADJUSTMENT FOR FIRST ORDER AUTOCORRELATION AND COMPANY DUMMIES. COEFFICIENTS OF THE COMPANY DUMMIES ARE NOT REPORTED

Dependent Variable is $\frac{CS}{TA} - r \frac{CS}{TA}_{\text{lagged}}$

Independent Variables	r=0.0	r=0.5	r=0.7	r=0.8	r=0.9	r=1.0
$\frac{R}{TA} - r \frac{R}{TA}_{\text{lagged}}$	.5048 (26.32)	.5864 (25.74)	.6525 (26.07)	.6804 (26.25)	.6961 (26.36)	.6996 (26.55)
$YC - rYC_{\text{lagged}}$	.00305 (11.62)	.00265 (7.65)	.00208 (4.65)	.00153 (2.93)	.000687 (1.13)	-.000332 (-.49)
Standard Error of the Estimate	.0130	.0101	.0098	.0099	.0102	.0105
R^2	.8997	.8060	.6679	.5588	.4511	.4025
$\bar{R}^2$	.8919	.7909	.6420	.5244	.4083	.3560
F-Ratio (93,1194 d.f.)	115.13	53.34	25.82	16.26	10.55	8.65

TABLE 4-15-e

CROSS-SECTIONAL/TIME-SERIES REGRESSIONS INCORPORATING AN ADJUSTMENT FOR FIRST ORDER AUTOCORRELATION AND COMPANY DUMMIES. COEFFICIENTS OF THE COMPANY DUMMIES ARE NOT REPORTED

	Dependent Variable is $\frac{PS}{TA} - r(\frac{PS}{TA})_{\text{lagged}}$					
Independent Variables	r=0.0	r=0.5	r=0.7	r=0.8	r=0.9	r=1.0
$\frac{R}{TA} - r(\frac{R}{TA})_{\text{lagged}}$	.1496 (10.22)	.1036 (7.49)	.0632 (4.91)	.0418 (3.40)	.0250 (2.12)	.0181 (1.59)
$YC - rYC_{\text{lagged}}$	.00205 (10.21)	.00173 (8.22)	.00114 (4.94)	.000596 (2.40)	-.000080 (-.30)	-.000647 (-2.22)
Standard Error of the Estimate	.0099	.0061	.0051	.0047	.0045	.0046
R^2	.7473	.6872	.5802	.4707	.3117	.1589
$\bar{R}^2$	.7277	.6629	.5476	.4294	.2581	.0933
F-Ratio (93,1194 d.f.)	37.97	28.21	17.75	11.42	5.81	2.42

TABLE 4-15-f

CROSS-SECTIONAL/TIME-SERIES REGRESSIONS INCORPORATING AN ADJUSTMENT FOR FIRST ORDER AUTOCORRELATION AND COMPANY DUMMIES. COEFFICIENTS OF THE COMPANY DUMMIES ARE NOT REPORTED

	Dependent Variable is $\frac{TS}{TA} - r(\frac{TS}{TA})_{\text{lagged}}$					
Independent Variables	r=0.0	r=0.5	r=0.7	r=0.8	r=0.9	r=1.0
$\frac{R}{TA} - r(\frac{R}{TA})_{\text{lagged}}$	.6505 (24.69)	.6887 (24.80)	.7164 (25.06)	.7238 (25.10)	.7233 (25.11)	.7200 (25.32)
$YC - rYC_{\text{lagged}}$	.00506 (14.03)	.00436 (10.32)	.00323 (6.32)	.00216 (3.71)	.000670 (1.01)	-.000896 (-1.23)
Standard Error of the Estimate	.0179	.0123	.0112	.0110	.0111	.0114
R^2	.8805	.8125	.6987	.5943	.4729	.4036
$\bar{R}^2$	.8711	.7979	.6752	.5627	.4318	.3572
F-Ratio (93,1194 d.f.)	94.56	55.63	29.77	18.81	11.52	8.69

$\frac{R^*}{TA}$ remain highly significant for $\frac{CS^*}{TA}$ and $\frac{TS^*}{TA}$, but the significance is lowered for $\frac{PS^*}{TA}$. The coefficients of YC* change from significantly positive to insignificantly negative as ρ is increased.

To summarize, the above model is used to adjust for two basic problems in pooling the cross-sectional/time-series data: the problem of serial correlation and the problem of non-homogeneous firm intercepts in the cross sections. With this model, the standard errors and coefficients of determination improved substantially. The model continues to highlight the significant role of surplus in determining asset structure: the relative amount of surplus remains negatively related to the relative amount of bonds and mortgages the life insurance company buys and positively related to the proportion of equities owned. The yield level proxy is significantly negatively related to the amount of bonds and mortgages owned, but this variable has less significance in the equity equations.

In Table 4-16, the LSDV model with adjustments for first-order autocorrelation is also used to estimate equations for bonds and mortgages similar to the first two of Table 4-15, except the yield ratio $\frac{YM}{YC}$ is used instead of the yield level. In the basic OLS model using relative yields in Table 4-10, it was found that both $\frac{B}{TA}$ and $\frac{M}{TA}$ were positively related to $\frac{YM}{YC}$, an unsatisfactory result if bonds and mortgages are assumed to be substitutes.

TABLE 4-16-a

CROSS-SECTIONAL/TIME-SERIES REGRESSIONS INCORPORATING AN ADJUSTMENT FOR FIRST ORDER AUTOCORRELATION AND COMPANY DUMMIES. COEFFICIENTS OF THE COMPANY DUMMIES ARE NOT REPORTED

	Dependent Variable is $\frac{B}{TA} - r(\frac{B}{TA})_{\text{lagged}}$					
Independent Variables	r=0.0	r=0.5	r=0.7	r=0.8	r=0.9	r=1.0
$\frac{R}{TA} - r(\frac{R}{TA})_{\text{lagged}}$	-.06067 (-1.10)	-.1381 (-2.74)	-.1580 (-3.46)	-.1582 (-3.67)	-.1529 (-3.72)	-.1465 (-3.64)
$\frac{YM}{YC} - r(\frac{YM}{YC})_{\text{lagged}}$	.1429 (17.19)	.07666 (11.99)	.03958 (6.81)	.01804 (3.15)	-.005589 (-.95)	-.03104 (-4.87)
Standard Error of the Estimate	.0373	.0224	.0181	.0167	.0162	.0164
R^2	.8201	.7468	.6024	.4306	.2013	.1714
$\bar{R}^2$	.8061	.7271	.5714	.3862	.1391	.1069
F-Ratio (93,1194 d.f.)	58.52	37.87	19.45	9.71	3.24	2.66

TABLE 4-16-b

CROSS-SECTIONAL/TIME-SERIES REGRESSIONS INCORPORATING AN ADJUSTMENT FOR FIRST ORDER AUTOCORRELATION AND COMPNAY DUMMIES. COEFFICIENTS OF THE COMPANY DUMMIES ARE NOT REPORTED

	Dependent Variable is $\frac{M}{TA} - r(\frac{M}{TA})_{\text{lagged}}$					
Independent Variables	r=0.0	r-0.5	r=0.7	r=0.8	r=0.9	r=1.0
$\frac{R}{TA} - r(\frac{R}{TA})_{\text{lagged}}$	-.1291 (-2.43)	-.2320 (-4.88)	-.3351 (-7.86)	-.3890 (-9.69)	-.4312 (-11.26)	-.4547 (-12.12)
$\frac{YM}{YC} - r(\frac{YM}{YC})_{\text{lagged}}$	.07213 (9.01)	.08061 (13.36)	.08810 (16.21)	.09323 (17.47)	.09906 (18.07)	.1052 (17.70)
Standard Error of Estimate	.0359	.0211	.0169	.0156	.0150	.0153
R^2	.8576	.8146	.7234	.6100	.4321	.3421
$\bar{R}^2$	.8465	.8001	.7019	.5796	.3879	.2909
F-Ratio (93,1194 d.f.)	77.34	56.40	33.58	20.08	9.77	6.68

The summary statistics of the equations of Table 4-16 follow the same patterns as those of the same statistical model above: higher coefficients of determination and smaller standard errors. For high ρ's, the mortgage equations in Table 4-16-b have much higher coefficients of determination than they did in Table 4-15-b using yield levels. Surplus remains negatively related to bond and mortgage holdings, and, as the adjustment factor ρ is increased, the coefficients of surplus become more highly significant.

As ρ is increased, the coefficients of $\frac{YM^*}{YC}$ increase in size and significance for mortgages. Interestingly, the coefficients of $\frac{YM^*}{YC}$ decrease in size and become negative as ρ is increased in the bond equation. An increase in the yield on mortgages relative to bonds appears to result in greater investments in mortgages. The effect of this same increase on bonds in Table 4-16-a would be to decrease bondholdings if the appropriate equation had a ρ slightly greater than 0.9. Where the OLS model exhibited positive coefficients for both bonds and mortgages, the LSDV model yielded the desired positive coefficient for mortgages and no significant relationship for bonds. The demand for bonds as a function of $\frac{YM}{YC}$ appears to be sensitive to changes in the adjustment factor ρ.

To summarize the results of the LSDV model with adjustments for autocorrelation, the overall equations were improved and the hypotheses derived from the chance-constrained model were substantiated with two qualifications: (1) the yield level was found to be a less significant variable in explaining the demand for equities than in the simpler regression model of the previous section, and (2) the yield ratio $\frac{YM}{YC}$ had an ambiguous impact on the demand for bonds. These results may be due to the small number of time periods or to dynamics of the investment process in life insurance companies for which the model may be inappropriate.

Cross-Sectional/Time-Series Regressions Using Distributed Lag Models

This section estimates the demand for various types of securities using the Koyck and the Almon distributed lag models, both of which provide insight into the dynamics of the portfolio adjustment process. The Koyck model assumes a common lag structure for all independent variables and shows the speed of adjustment to exogenous changes. Since serial correlation can bias the results of the Koyck model, the estimating equation used here removes the effects of first-order autocorrelation. The Almon technique allows more general distributed lag structures and a different structure for each independent variable, thus permitting dynamic comparisons of the impacts of the exogenous variables.

Koyck distributed lag model

This model is frequently used when the effect of the independent variable declines exponentially with time or when a simple stock-adjustment process exists. A company's current portfolio mixture is the result of current and past portfolio decisions. The capacity to change the portfolio is based on the gross operating and investment cash inflow to the firm and potential liquidation of marketable securities. The rate of utilization of this capacity to change the portfolio is affected by the nature of the investment process described in Chapter II.

An assumption that the dependent variable is determined by the current and all previous values of the independent variables where the impact of the independent variables decays exponentially through time leads to the Koyck model.[16] The coefficients of the independent variables are assumed to decline geometrically as the lag increases:

$$B_k = B\lambda^k \qquad K = 0,\ 1,\ \ldots$$

where $0<\lambda<1$ and k is the lag in time periods. The distributed lag model is

$$Y_t = BX_t + \lambda BX_{t-1} + \lambda^2 BX_{t-2} + \cdots + E_t$$

The difference between the above equation and the result of lagging the equation one period and multiplying it by the scalar λ yields the estimating equation.

$$\lambda Y_{t-1} = \lambda BX_{t-1} + \lambda^2 BX_{t-2} + \cdots + \lambda E_{t-1}$$

$$Y_t = BX_t + \lambda Y_{t-1} + E_t^*$$

where

$$E_t^* = E_t - \lambda E_{t-1}$$

---

[16] Johnston, Econometric Methods, pp. 298-303, and Goldberger, Econometric Theory, pp. 274-78.

A second explanation for the Koyck model is the stock-adjustment model in which a fixed proportion of a disequilibrium is satisfied in a given period.[17] Let Y^d equal the desired and Y be the actual portfolio proportions. Then

$$Y_t - Y_{t-1} = \Delta Y = \delta(Y_t^d - Y_{t-1}) + E_t$$
$$Y_t^d = a + bX$$

This is easily estimated as

$$Y = \delta a + \delta bX + (1-\delta)\ Y_{t-1} + E_t \qquad .$$

The stock adjustment coefficient δ is the complement of the coefficient estimated for the lagged dependent variable.

One complication which must be dealt with here is the presence of serial correlation, which in a Koyck model leads to biased and inconsistent results. Griliches points out that simple autocorrelation can be confused with the Koyck model.[18] Consider the following model:

$$Y_t = aX_t + u_t$$

where

$$u_t = \rho u_{t-1} + e_t$$

If we mistakenly estimate a Koyck model using OLS

$$Y_t = aX_t + bY_{t-1} + v_t$$

the coefficient of Y_{t-1} will approximately equal ρ because the model has been misspecified. The correct specification

---

[17] Goldberger, <u>Econometric Theory</u>, pp. 274-5.

[18] Zvi Griliches, "Distributed Lags: A Survey," <u>Econometrica</u>, XXXV (January, 1967), 33-4.

would be

$$Y_t = aX_t + \rho Y_{t-1} - a\rho X_{t-1} + e_t$$

Obviously, if this equation is estimated and the coefficient of X_{t-1} equals minus the product of the coefficients of X_t and Y_{t-1}, the serial correlation model would appear to be the correct model and not the Koyck model.

The results of estimating this equation with company intercepts for the six dependent variables are given in Table 4-17. The products of the coefficients of current independent variable and the lagged dependent variable are in the neighborhood of the coefficients of the lagged dependent variables. The estimates for ρ range from .664 to .915. Since the autocorrelation biases the coefficients of the lagged dependent variable, the Koyck model cannot be estimated using OLS.

It is possible to estimate the Koyck distributed lag model with first-order autocorrelation with this equation:[19]

$$Y_t = aX_t - a\rho X_{t-1} + (\rho+b)\ Y_{t-1} - b\rho Y_{t-1} + e_t$$

or

$$Y_t - \rho Y_{t-1} = a(X_t - \rho X_{t-1}) + b(Y_{t-1} - \rho Y_{t-2}) + e_t$$

This equation is estimated by iterating over ρ in Table 4-18 in which company dummies are included. Surplus and the yield level remain negatively related to the demand for bonds, mortgages, and fixed assets, and surplus is

---

[19] Ibid., 40.

TABLE 4-17

EXHIBIT OF SERIAL CORRELATION BIAS IN KOYCK MODEL

Independent Variables	Dependent Variables					
	$\frac{B}{TA}$	$\frac{M}{TA}$	$\frac{F}{TA}$	$\frac{CS}{TA}$	$\frac{PS}{TA}$	$\frac{TS}{TA}$
$\frac{R}{TA}$	-.212257 (-5.08)	-.386225 (-9.77)	-.600959 (-15.53)	.674981 (27.24)	.031146 (2.65)	.721355 (25.72)
YC	-.005738 (-5.48)	-.000213 (-.22)	-.006331 (-6.64)	-.000309 (-.50)	-.000638 (-2.20)	-.000784 (-1.13)
Dep. Var._{-1}	.905875 (74.51)	.876152 (70.54)	.888746 (58.84)	.664313 (31.36)	.915005 (67.95)	.805422 (45.57)
$(\frac{R}{TA})_{-1}$	.180829 (4.29)	.405544 (10.10)	.590310 (14.89)	-.538798 (-19.36)	-.001170 (-10)	-.611497 (-19.86)
YC_{-1}	.006282 (5.60)	-.005933 (-5.49)	.000580 (.55)	.002459 (3.69)	.000887 (2.81)	.003028 (4.01)
Std. Error of Estimate	.0160	.0151	.0147	.0094	.0045	.0107
R^2	.9672	.9750	.9683	.9473	.9488	.9572
$\bar{R}^2$	.9646	.9730	.9657	.0430	.9447	.9538
F-Ratio	365.80	484.81	378.43	222.95	229.97	277.67

TABLE 4-18-a

KOYCK MODEL USING COMPANY DUMMIES AND AUTOCORRELATION ADJUSTMENT

Independent Variables	Dependent Variable is $\frac{B}{TA} - r(\frac{B}{TA})_{-1}$				
	r=0.5	r=0.7	r=0.8	r=0.9	r=1.0
$\frac{R}{TA} - r(\frac{R}{TA})_{-1}$	-.138546 (-3.78)	-.195553 (-4.48)	-.216892 (-5.31)	-.230503 (-5.65)	-.246411 (-6.04)
$YC - rYC_{-1}$	-.001671 (-2.78)	-.003421 (-4.53)	-.003764 (-4.47)	-.003550 (-3.77)	-.003691 (-3.54)
$(\frac{B}{TA})_{-1} - r(\frac{B}{TA})_{-2}$	.675598 (31.36)	.433621 (16.58)	.295275 (10.61)	.92374 (6.48)	.168592 (5.71)
Standard Error of Estimate	.0155	.0154	.0153	.0153	.0160
R^2	.8739	.7032	.5148	.2630	.1977
$\bar{R}^2$	.8631	.6778	.4733	.2001	.1292
F-Ratio	81.17	27.75	12.43	4.18	2.89

TABLE 4-18-b

KOYCK MODEL USING COMPANY DUMMIES AND AUTOCORRELATION ADJUSTMENT

	Dependent Variable is $\frac{M}{TA} - r(\frac{M}{TA})_{-1}$				
Independent Variables	r=0.5	r=0.7	r=0.8	r=0.9	r-1.0
$\frac{R}{TA} - r(\frac{R}{TA})_{-1}$	-.298803 (-8.27)	-.416930 (-10.26)	-.445310 (-10.64)	-.442306 (-10.45)	-.415889 (-9.90)
$YC - rYC_{-1}$	-.006570 (-11.72)	-.007266 (-9.75)	-.006947 (-8.07)	-.005425 (-5.55)	-.002618 (-2.46)
$(\frac{M}{TA})_{-1} - r(\frac{M}{TA})_{-2}$	.698362 (30.69)	.513026 (18.61)	.414406 (14.18)	.343283 (11.48)	.315695 (10.65)
Standard Error of Estimate	.0153	.0154	.0156	.0158	.0163
R^2	.9028	.7718	.6172	.3766	.2479
$\bar{R}^2$	.8945	.7523	.5846	.3234	.1837
F-Ratio	108.78	39.62	18.89	7.08	3.86

TABLE 4-18-c

KOYCK MODEL USING COMPANY DUMMIES AND AUTOCORRELATION ADJUSTMENT

	Dependent Variable is $\frac{F}{TA} - r(\frac{F}{TA})_{-1}$			
Independent Variables	r=0.7	r=0.8	r=0.9	r=1.0
$\frac{R}{TA} - r(\frac{R}{TA})_{-1}$	-.601070 (-14.78)	-.644194 (-15.63)	-.646566 (-15.86	-.621733 (-15.79)
$YC - rYC_{-1}$	-.011618 (-14.69)	-.011329 (-13.10)	-.009524 (-10.13)	-.006595 (-6.65)
$(\frac{F}{TA})_{-1} - r(\frac{F}{TA})_{-2}$	.379938 (14.61)	.276517 (10.38)	.174948 (6.48)	.086849 (3.22)
Standard Error of Estimate	.0155	.0153	.0152	.0153
R^2	.7307	.5853	.3823	.2635
$\bar{R}^2$	.7077	.5499	.3296	.2006
F-Ratio	31.77	16.53	7.25	4.19

TABLE 4-18-d

KOYCK MODEL USING COMPANY DUMMIES AND AUTOCORRELATION ADJUSTMENT

	Dependent Variable is $\frac{CS}{TA} - r(\frac{CS}{TA})_{-1}$				
Independent Variables	r=0.5	r=0.7	r=0.8	r=0.9	r=1.0
$\frac{R}{TA} - r(\frac{R}{TA})_{-1}$	.606578 (25.55)	.678788 (27.02)	.694353 (27.04)	.697298 (26.88)	.694080 (26.90)
$YC - rYC_{-1}$	.002626 (7.31)	.002302 (5.04)	.001682 (3.20)	.000700 (1.17)	-.000386 (-.59_
$(\frac{CS}{TA})_{-1} - r(\frac{CS}{TA})_{-2}$	.039936 (1.65)	-.086439 (-3.72)	-.126234 (-5.53)	-.150889 (-6.72)	-.160360 (-7.26)
Standard Error of Estimate	.0099	.0096	.0096	.0097	.0101
R^2	.8116	.6767	.5761	.4839	.4537
$\bar{R}^2$	.7955	.6491	.5399	.4399	.4070
F-Ratio	50.46	24.52	15.92	10.98	9.73

TABLE 4-18-e

KOYCK MODEL USING COMPANY DUMMIES AND AUTOCORRELATION ADJUSTMENT

	Dependent Variable is $\frac{PS}{TA} - r(\frac{PS}{TA})_{-1}$				
Independent Variables	r=0.5	r=0.7	r=0.8	r=0.9	r=1.0
$\frac{R}{TA} - r(\frac{R}{TA})_{-1}$	.039824 (3.83)	.037347 (3.24)	.031887 (2.66)	.023843 (2.04)	.020570 (1.78)
$YC - rYC_{-1}$	.000308 (1.91)	.000255 (1.21)	.000010 (.04)	-.000414 (-1.53)	-.000786 (-2.67)
$(\frac{PS}{TA})_{-1} - r(\frac{PS}{TA})_{-2}$	.694516 (30.06)	.494051 (17.42)	.372753 (12.23)	.274736 (8.67)	.239402 (7.48)
Standard Error of Estimate	.0043	.0044	.0044	.0044	.0045
R^2	.8510	.7057	.5723	.3903	.2194
$\bar{R}^2$	.8383	.6805	.5358	.3383	.1528
F-Ratio	66.90	28.08	15.67	7.50	3.29

TABLE 4-18-f

KOYCK MODEL USING COMPANY DUMMIES AND AUTOCORRELATION ADJUSTMENT

	Dependent Variable is $\frac{TS}{TA} - r(\frac{TS}{TA})_{-1}$				
Independent Variables	r=0.5	r=0.7	r=0.8	r=0.9	r=1.0
$\frac{R}{TA} - r(\frac{R}{TA})_{-1}$	.651837 (23.43)	.735184 (25.68)	.741983 (25.88)	.733009 (25.75)	.724490 (25.85)
$YC - rYC_{-1}$	.003361 (7.95)	.003007 (5.78)	.002022 (3.45)	.000463 (.70)	-.001095 (-1.54)
$(\frac{TS}{TA})_{-1} - r(\frac{TS}{TA})_{-2}$	.212891 (8.79)	.021402 (.89)	-.053307 (-2.24)	-.104267 (-4.47)	-.123940 (-5.43)
Standard Error of Estimate	.0115	.0109	.0107	.0107	.0110
R^2	.8377	.7154	.6146	.5051	.4507
$\bar{R}^2$	.8239	.6911	.5817	.4628	.4038
F-Ratio	60.47	29.44	18.68	11.95	9.61

positively related to the demand for common, preferred, and total stock. However, the estimated coefficients for YC in the equity equations and for the lagged dependent variable in all equations appear to be very sensitive to the value of ρ used. The Hildreth-Lu minimum standard error criterion for selecting ρ is difficult to apply in this case with confidence because the standard errors change relatively little as ρ is iterated. Consequently, estimates for ρ found in Table 4-15 of the previous section are applied here. These are .90 for $\frac{B}{TA}$, .87 for $\frac{M}{TA}$, .93 for $\frac{F}{TA}$, .73 for $\frac{CS}{TA}$, .93 for $\frac{PS}{TA}$, and .83 for $\frac{TS}{TA}$. With these estimates for ρ, it appears in Table 4-18 that the yield level is positively related to the demand for common stock and total equities and negatively related to the demand for preferred stock.

The coefficients of the lagged dependent variables indicate the persistence of the exponentially declining impact of the independent variables on alternately, the speed of adjustment to a disequilibrium. Using the values of ρ, the approximate values of λ and δ are:

	λ	δ
$\frac{B}{TA}$	.19	.81
$\frac{M}{TA}$	.36	.64
$\frac{F}{TA}$	.15	.85
$\frac{CS}{TA}$	0	1
$\frac{PS}{TA}$	.26	.74
$\frac{TS}{TA}$	0	1

Overall, these values imply that portfolio adjustments occur rapidly and that the impact of lagged independent variables declines rapidly.[20] More rapid speeds of adjustment for financial institutions compared to industrial firms should be possible because of the differences between financial assets and tangible assets outlined in Chapter I. Life insurance companies appear to make decisions based on current (as opposed to lagged) exogenous events and to affect the bulk of their desired portfolio adjustments within one period.

Almon distributed lag model

The nature of a lag structure in the life insurance investment process is affected by the time necessary to perceive changes in the independent variables, by the forward commitment procedures for mortgages and bonds, and by the presence of illiquid assets which cannot be converted to another form at a low cost. The high positive serial correlation of several estimating models in earlier parts of this chapter and the results of the Koyck distributed lag model argue for the existence of a lag structure which can be

---

[20]This is in marked contrast to the biased estimates obtained for the Koyck model using OLS given in Appendix IV-7. The coefficients for the lagged dependent variables were .91 for $\frac{B}{TA}$, .99 for $\frac{M}{TA}$, .95 for $\frac{F}{TA}$, .48 for $\frac{CS}{TA}$, .91 for $\frac{PS}{TA}$, and .66 for $\frac{TS}{TA}$. The economic interpretation of these biased coefficients would be quite different from those obtained from the Koyck model, correctly specified.

explored further with the Almon lag technique.[21]

Almon Lags have some important advantages over the Koyck model presented above. The Almon lag structure can assume a general polynomial shape and is not restricted to the geometric structure of the Koyck lag. Separate structures may be estimated for each independent variable. Finally, as established in the previous section, the presence of autocorrelation leads to biased estimates in a Koyck model: but serial correlation results in inefficiency, but not bias, in the Almon model.

In the Almon lag model, the weights of the current and lagged independent variables are assumed to lie on a polynomial of degree ρ:

$$Y_t = w_o X_t + w_1 X_{t-1} + \cdots + w_n X_{t-n} + E_t$$

where

$$w_i = \lambda_o + \lambda_1 i + \lambda_2 i^2 + \cdots + \lambda_p i^p$$

$$i = 0, 1, \ldots, n; \qquad p \leq n$$

This *a priori* assumption about the weights leads to more efficient estimates and more powerful tests. In addition, it is possible to constrain the coefficients of the independent variable for period $t - n$ or period $t + 1$ to zero. The researcher applying the Almon lag technique must choose the length of the lag n and the degree of the polynomial p and

---

[21] Shirley Almon, "The Distributed Lag Between Capital Appropriations and Expenditures," *Econometrica*, XXXIII (January, 1965), 178-96.

and he must make appropriate assumptions about endpoint constraints. Of course, if a specification error is committed, the estimates and tests will be invalid.

Tables 4-19 and 4-20 present equations estimated using the Almon lag technique with a second degree polynomial, n = 4 for yields, n = 3 for surplus, and the coefficients at (t-n) are constrained to zero. This specification led to generally smaller standard errors than several alternative specifications which were tried. Since the data set contains 15 annual observations for 92 firms, the four year lag reduced the annual observations to 11 per company and, in turn, the degree of the polynomial is constrained by the lag length, $\rho<n$. Since the coefficients of the independent variables to which the Almon lag was applied were extremely sensitive to the assumed lag structure and since there is no *a priori* correct structure, the specific results of Tables 4-19 and 4-20 should not be overstated.

The coefficients and significance of the stock dummy, the New York license dummy, and the New York charter dummy are in accordance with those found in prior models of this chapter, even though the estimates of Tables 4-19 and 4-20 are based on only eleven annual observations per company. Consequently, it is only necessary to reaffirm that the interpretations previously applied to these dummy variables would be the same for the Almon lag model. In addition, the sign and significance of the sum of the coefficients of

TABLE 4-19

CROSS-SECTIONAL/TIME-SERIES RESULTS USING THE ALMON LAG TECHNIQUE WITH YIELD LEVELS

	Dependent Variable		
Independent Variables	$\frac{B}{TA}$	$\frac{M}{TA}$	$\frac{F}{TA}$
CONSTANT	.4228	.5811	1.0039
YC(t)	-.0151 (-3.97)	.0094 (2.22)	-.0057 (-1.80)
(t-1)	-.0009 (-.92)	-.0065 (-6.04)	-.0074 (-9.09)
(t-2)	.0064 (2.19)	-.0134 (-4.17)	-.0070 (-2.90)
(t-3)	.0067 (2.54)	-.0112 (-3.86)	-.0046 (-2.08)
(t-4)	0	0	0
$\frac{R}{TA}$(t)	-.3417 (-2.10)	-.1167 (-.65)	-.4584 (-3.37)
(t-1)	.0118 (.20)	-.2688 (-4.17)	-.2569 (-5.28)
(t-2)	.1258 (1.15)	-.2299 (-1.89)	-.1041 (-1.13)
(t-3)	0	0	0
SD	.0039 (.65)	.0389 (5.81)	.0428 (8.46)
NL	.0404 (7.37)	-.0554 (-9.12)	-.0150 (-3.26)
NC	.0072 (.84)	.0212 (2.24)	.0284 (3.97)
SUM YC	-.0030 (-.92)	-.0218 (-6.04)	-.0248 (-9.09)
SUM $\frac{R}{TA}$	-.2041 (-4.28)	-.6154 (-11.62)	-.8195 (-20.48)
Standard Error	.0740	.0821	.0620
R^2	.1402	.1899	.3853
Degrees of Freedom	1004	1004	1004

TABLE 4-19--Continued

Independent Variable	Dependent Variable $\frac{B}{TA}$	$\frac{M}{TA}$	$\frac{F}{TA}$
CONSTANT	-.0450	-.0159	-.0597
YC(t)	-.0002 (-.13)	.0000 (.05)	-.0002 (-09)
(t-1)	.0015 (4.08)	.0005 (2.41)	.0020 (4.73)
(t-2)	.0021 (1.92)	.0007 (1.05)	.0028 (2.19)
(t-3)	.0016 (1.61)	.0005 (.86)	.0021 (1.83)
(t-4)	0	0	0
$\frac{R}{TA}$(t)	.5071 (8.20)	.0008 (.02)	.5059 (7.05)
(t-1)	.0888 (4.02)	.1002 (7.48)	.1895 (7.38)
(t-2)	-.0802 (-1.92)	.0999 (3.95)	.0208 (.43)
(t-3)	0	0	0
SD	-.0149 (-6.49)	-.0017)=1.24)	-.0172 (-6.43)
NL	.0133 (6.35)	.0064 (5.03)	.0189 (7.80)
NC	-.0185 (-5.37)	.0001 (.07)	-.0174 (-4.62)
SUM YC	.0051 (4.08)	.0018 (2.41)	.0068 (4.73)
SUM $\frac{R}{TA}$	.5157 (28.38)	.2009 (18.25)	.7162 (33.94)
Standard Error	.0282	.0171	.0327
R^2	.5058	.3018	.5900
Degrees of Freedom	1004	1004	1004

TABLE 4-20

CROSS-SECTIONAL/TIME-SERIES RESULTS USING THE ALMON LAG TECHNIQUE AND YIELD RATIOS

Independent Variables	Dependent Variable $\frac{B}{TA}$	$\frac{M}{TA}$
CONSTANT	.2999	.2324
$\frac{YM}{YC}(t)$	.1488 (3.60)	-.1569 (-3.42)
(t-1)	.0263 (3.13)	.0578 (6.20)
(t-2)	-.0394 (-1.45)	.1555 (5.15)
(t-3)	-.0481 (-1.91)	.1362 (4.86)
(t-4)	0	0
$\frac{R}{TA}(t)$	-.2813 (-1.75)	-.2032 (-1.14)
(t-1)	-.0066 (-.12)	-.2396 (-3.76)
(t-2)	.0871 (.80)	-.1718 (-1.43)
(t-3)	0	0
SD	.0035 (.58)	.0391 (5.84)
NL	.0406 (7.42)	-.0556 (-9.14)
NC	.0070 (.82)	.0215 (2.27)
SUM $\frac{YM}{YC}$	.0875 (3.13)	.1926 (6.20)
SUM $\frac{R}{TA}$	-.2008 (-4.21)	-.6146 (-11.60)
Standard Error	.0739	.0821
R^2	.1434	.1893
Degree of Freedom	1004	1004

YC, $\frac{R}{TA}$, and $\frac{YM}{YC}$ are generally similar to comparable earlier models where these variables appeared only contemporaneously with the dependent variables with two exceptions: the yield level proxy is less significant than in Table 4-4 except for mortgages, and the yield ratio is more significant for mortgages in Table 4-20 whereas it was more significant for bonds in Table 4-10. The standard errors are now lower for bonds and mortgages; otherwise these summary statistics were basically identical. Overall, the Almon lag model does not alter earlier results of the basic ordinary least squares model and it provides additional information about the dynamic impacts of yields and surplus on portfolio choices of life insurance companies.

As best seen in the equations for $\frac{F}{TA}$ and $\frac{CS}{TA}$ in Table 4-20, the impact of changes in surplus appears to be fairly rapid, mostly occurring in the current year. The role of changes in the yield level is relatively slow, with the current yield level usually less significant than the lagged values. Surplus and the yield level affect $\frac{B}{TA}$ faster than mortgages. The yield level has a weak influence on $\frac{PS}{TA}$ and surplus affects $\frac{PS}{TA}$ only when lagged.

In Table 4-20 where the yield ratio $\frac{YM}{YC}$ is substituted for the yield level, the coefficient of $\frac{YM}{YC}$ should be positive for $\frac{M}{TA}$ and negative for $\frac{B}{TA}$ if bonds and mortgages are substitutes. This coefficient is significantly negative for $\frac{M}{TA}$ for periods t-1 t-2, and t-3 and the coefficient has the wrong sign at the .01 significance level for mortgages in period t and for bonds at t and t-1. The coefficient has the desired negative sign for bonds in periods t-2 and t-3, but not at the .01 significance level. These results would be consistent with a hypothesis (a) that there are significant lags of at least a year in the investment response of mortgages and (b) that bonds are a residual category of assets which absorb investible funds when the life insurance company has nothing better to do with them. However, the empirical results are neither clear enough to support this hypothesis nor to reject the assumption that bonds and mortgages are good substitutes.

Briefly, both distributed lag models employed here show that portfolio adjustments take place mostly within a year, with the mortgage and preferred stock adjustments being the slowest. Adjustments to changes in equity are much faster than to changes in yields. The differences in the polynomial lag structures for bonds and mortgages may be due to the nature of the forward commitment process in the mortgage market and to the historical lack of an effective secondary mortgage market. The significance of coefficients of the sum of YC and $\frac{YM}{YC}$, the sum or $\frac{R}{TA}$, and NL, NC, and SD are basically identical to the results of the OLS or LSDV models. Both the Almon and Koyck models yield results that are consistent with the static chance-constrained model of Chapter III and conform to the dynamics expected from the institutional characteristics of the capital markets described in

Chapter II.

Summary

The empirical results of this chapter are in general, though not complete, agreement with the hypotheses generated by the chance-constrained model of life insurance portfolio behavior.

The relative quantity of surplus, $\frac{R}{TA}$, was consistently highly significant and had the correct signs for every type of econometric model used. Even with the introduction of company dummies, adjustments for serial correlation, and the use of distributed lags, surplus remained a highly significant determinant of life insurance portfolio choices. The hypothesis that a high amount of surplus provides a safety margin which allows life insurance companies to invest more aggressively in risky types of assets and yet to maintain a high probability of solvency is supported by the empirical behavior of 92 companies from 1957-71.

The performance of the yield variables was mixed, though generally conforming with the stated hypotheses. The earnings cushion provided by a high level of financial yields did correlate positively to investments in risky types of assets and negatively to investments in safer assets. An exception to this was the demand for equities in the models which removed autocorrelation in which the relationship between YC and $\frac{CS}{TA}$, $\frac{PS}{TA}$, and $\frac{TS}{TA}$ was not significant. Relative yields expressed as yield ratios had the predicted signs for mortgages and incorrect signs for preferred stock, and vacillated in the bond equations. Classical statistical problems of autocorrelation, multicollinearity, and small sample size (15 annual observations) could have caused these results. However, as will be outlined in the concluding chapter, this study has advanced questions about the theoretical and normative meaning of the yield variables, which has implications for financial econometric research in other areas.

Stock companies possessed the more conservative portfolios predicted because of the differences between policies sold by stock and mutual companies. Differences in investments of companies subject to and not subject to New York regulations were observed. However, since considerable slack in restricted investment categories was found for most companies, New York portfolio limitations cannot be considered generally binding.

Differences in portfolio choices based on company size were found: equities were found to be superior investments and, surprisingly, mortgages were found to be inferior to bonds. Canadian companies made statistically significant different portfolio choices from U.S. firms, but the absolute differences were not great.

Distributed lag models showed that response to changes in independent variables occurred very fast, particularly to changes in surplus. The dynamic results were reasonably consistent with the institutional features of the different capital markets.

Overall, the econometric results of this chapter sustained the predictions of the chance-constrained portfolio model. The economic implications of this research are discussed in the concluding chapter.

CHAPTER V

CONCLUSION

Based on the institutional and economic environment in which life insurance companies operate, this study developed a theoretical portfolio model with sufficient empirical content to yield readily testable hypotheses about life insurance portfolio behavior. The results have significant implications for financial research and provide considerable insight into life insurance investment choices.

State quantitative and qualitative restrictions on portfolio composition, the accounting procedures promulgated by the National Association of Insurance Commissioners, and Federal tax laws can influence the portfolio choices made by life insurance companies. Given these, the solvency of a life insurance company depends on its investment experience, its operating expenses, and the obligations resulting from its outstanding policies. The chance-constrained model developed in this study integrated the basic economic variables affecting life insurance portfolio management and suggested some economic relationships which were explored with various econometric techniques. In their role as one of the largest financial intermediaries, life insurance companies have substantial obligations to millions of households, are major suppliers of funds to two major capital markets, the corporate bond and mortgage markets, and are lessor sources of funds to other markets. Consequently, the life insurance industry directly affects the sectors of the economy with which it deals and is of interest to those who regulate and tax the industry.

Some inferences about the empirical performance of large life insurance companies are possible from this study. The variation in portfolio choices can be compartmentalized as follows: variation explained by the economic characteristics of a particular firm, variation explained by economic or regulatory forces external to the firm, and, finally, variation which is not explained by any variables used here.

Variables endogenous to the firm proved to be highly significant determinants of investment choices. The amount of surplus was positively related to investments in riskier types of assets, as predicted by the chance-constrained model, and stock companies tended to invest more conservatively than mutuals, also as predicted. While not derived from the chance-constrained model, company size was correlated positively with riskier types of investments. These results imply a rational economic basis for portfolio differences between firms.

Variables exogeneous to the firm proved less significant than endogeneous variables. While companies subject to New York regulations did possess lower percentages of assets invested in equities, *ceteris paribus*, the vast majority of these companies had considerable slack available in this investment category. Geographic preferences between bonds and mortgages appear to exist apart from regulations. However, portfolio choices proved relatively insensitive to yield variables.

Capital markets are considered to be allocationally efficient if resources flow to those uses promising the highest rates of return; the demand for various securities should be responsive to yield changes. The yield level was negatively related to the investment in fixed income securities, as predicted. Otherwise, the yield level and yield ratios failed to give consistent results. The statistical problems of multicollinearity, autocorrelation, and limited sample size (15 annual observations) were dealt with by limiting the hypotheses advanced and through the particular econometric techniques employed. In addition to these statistical problems, this study highlights three further difficulties which question the normative significance of yield variables. These statistical and economic problems could apply to other research areas, as well.

First, the assumption that the demand for a security should be positively related to its own yield, <u>ceteris paribus</u>, was found invalid. In a portfolio model in which there is a tradeoff between yield and probability of insolvency (or yield and risk), the demand for a security logically can be inversely related to its yield since an increased yield on a low return security increases the risk-taking capacity of a life insurance company, enabling it to shift towards riskier securities and maintain at least the same probability of solvency.

A second problem is that yield changes for a security on which a constraint is binding may have no impact on the demand for that security. Yield fluctuations below that point where the demand is zero have no impact on demand when non-negativity constraints apply, and, likewise, once an upper bound constraint is binding, further yield increases should have a completely inelastic demand response. In the portfolio model, high correlations among yields increase the likelihood that these upper and lower bound constraints are binding.

A third and final problem is that measured yield changes may not signify that the desirability of a security is changed. Yield differentials may change because the capital markets have perceived changes in the characteristics of a security or type of security. Relative yields are based on complex evaluations of risk of default, liquidity, transactions costs, callability, tax treatments, maturity, and inflationary expectations. A rational investor who sees a yield change in response to such economic characteristics could purchase more, the same, or less of such a security when these other characteristics are not quantified and his tradeoffs are not known.

The considerable between firm variation accounted for by the use of company dummies might be reduced by introducing additional explanatory variables beyond those used here. The lack of homogeneity among firms may also be caused by a high degree of substitutability among financial assets, which makes financial econometric analysis more challenging. If two securities are perfect substitutes, then the relative demands for them may be indeterminate for some parameter values. Efficient capital markets should price assets to remove strong preferences and to facilitate indifference across securities.

The dynamics of the investment process was explored in two distributed lag models. Most of the portfolio adjustment to changes in explanatory variables occurred within one year, with bonds and common stock responding the fastest, and changes in surplus resulted in portfolio shifts faster than changes in yield variables. Higher adjustment rates should be expected of financial intermediaries than for nonfinancial businesses.

The life insurance investment process is complex, dealing with the economics of the capital markets, portfolio theory, and the theory of financial intermediation on the one hand and varied state regulations, Federal tax laws, and accounting procedures on the other. Superimposed on this complex system, the chance-constrained portfolio model hypothesized some patterns of behavior which were verified with econometric analysis. These results should interest all parties affected by the life insurance industry.

APPENDICES

APPENDIX III

In Chapter III, the sensitivity of portfolio choices to the parameters of the chance-constrained model were discussed. This appendix gives a graphical example of each of the parameter changes mentioned in that chapter.

For simplicity, assume a two-security portfolio with a portfolio return and risk as follows:

$$R = X_1 r_1 + X_2 r_2$$

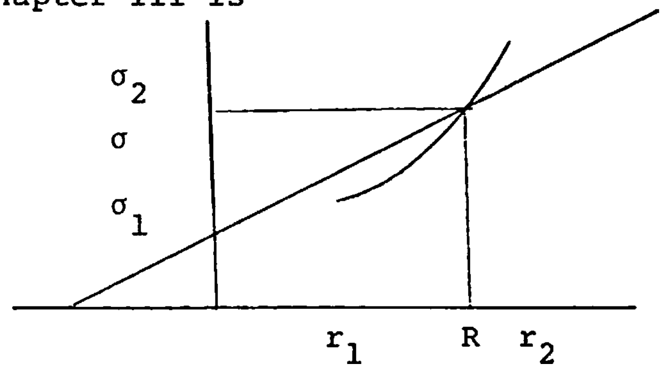
$$\sigma = (X_1^2\sigma_{11} + X_2^2\sigma 22 + 2X_1X_2\sigma_{12})^{1/2}$$

Let $r_1 < r_2$

$\sigma_{11} < \sigma_{22}$

Security 2 is the riskier, higher return security.

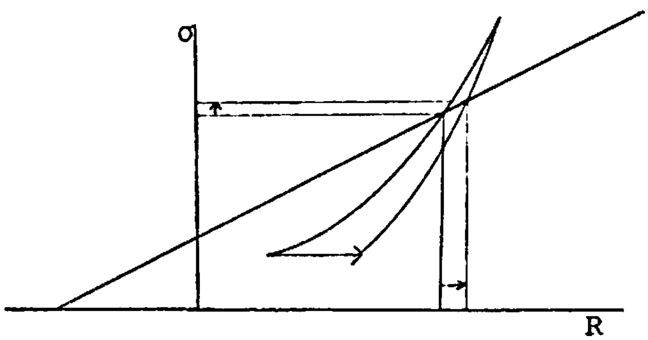
The chance-constrained model, as developed in Chapter III is



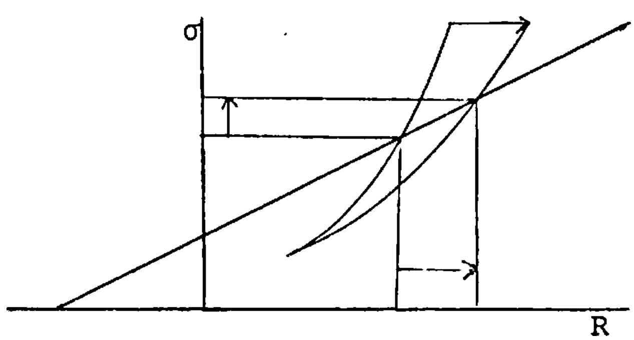
$$\sigma = \frac{1}{Z}(S-kLR) + \frac{1}{Z}r$$

Changes in r_1, r_2, σ_{11}, σ_{22}, σ_{12}, S, k, or Z will change the optimal (R,) combination and its underlying portfolio composition (X_1 and X_2). With r_1 and r_2 constant, a higher R can occur only with a shift from security 1 towards security 2, and, with σ_{11}, σ_{22}, and σ_{12} constant (and if the slope of the efficiency frontier is positive), a higher portfolio σ can occur only by increasing X_2 and decreasing X_1.

All of the parameter changes discussed in Chapter III which would cause the choice of a higher (R,σ) combination are illustrated in the following graphs and, in turn, summarized in Table III.

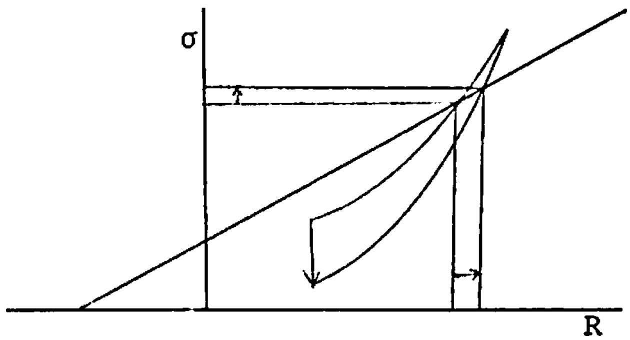


(1) increase in r_1

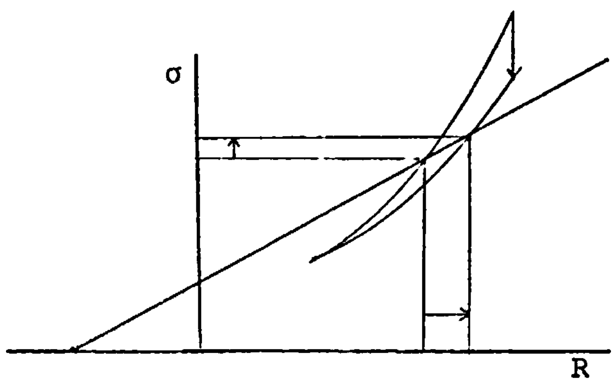


(2) increase in r_2

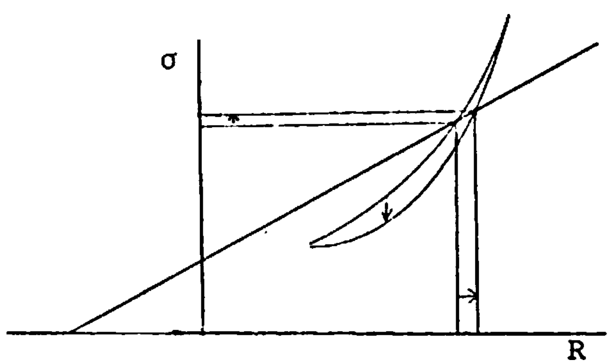
APPENDIX III--Continued



(3) decrease in σ_{11}

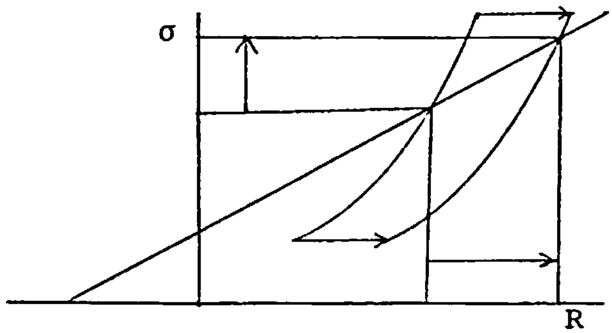


(4) decrease in σ_{22}

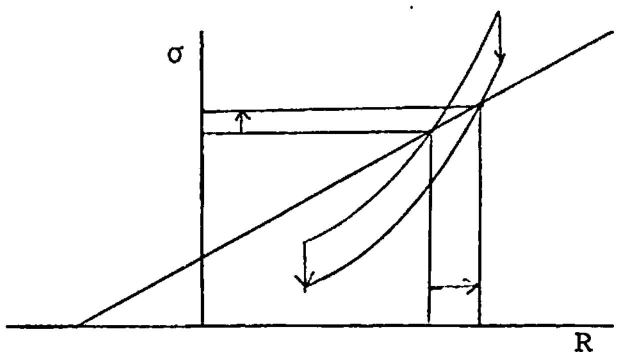


(5) decrease in σ_{12}

APPENDIX III--Continued

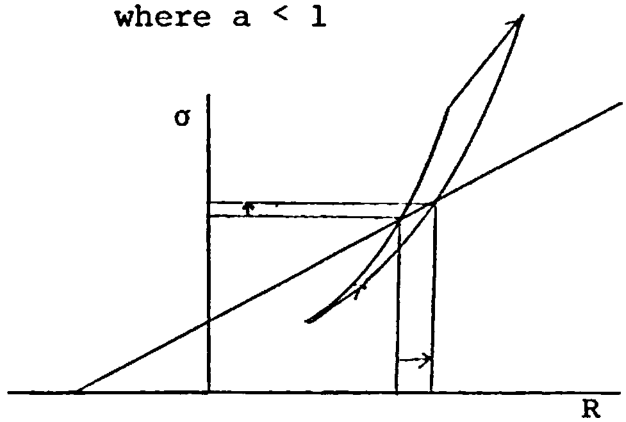


(6) yields increase by a constant
$r_1' = r_1 + k; \ r_2' = r_2 + k$



(7) Proportional decrease in square roots of elements of variance-covariance matrix
$\sigma_{11}' = a^2\sigma_{11}; \ \sigma_{22}' = a^2\sigma_{22}; \ \sigma_{12}' = a^2\sigma_{12}$

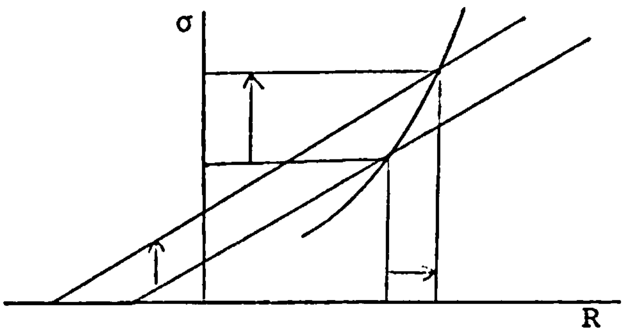
where $a < 1$



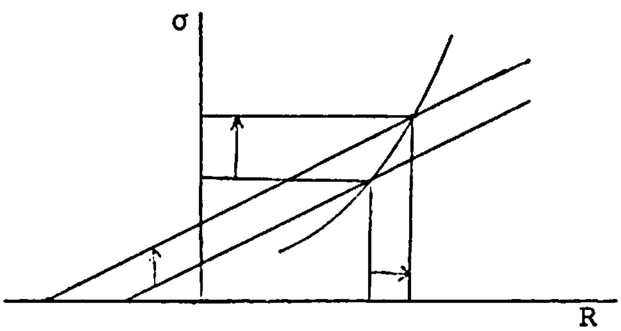
(8) Proportional increase in elements of return vector and square roots of elements of variance-covariance matrix
$r_1' = br_1; \ r_2' = br_2; \ \sigma_{11}' = b^2\sigma_{11}; \ \sigma_{22}' = b^2\sigma_{22};$

$\sigma_{12} = b^2 \ _{12}$ where $b > 1$

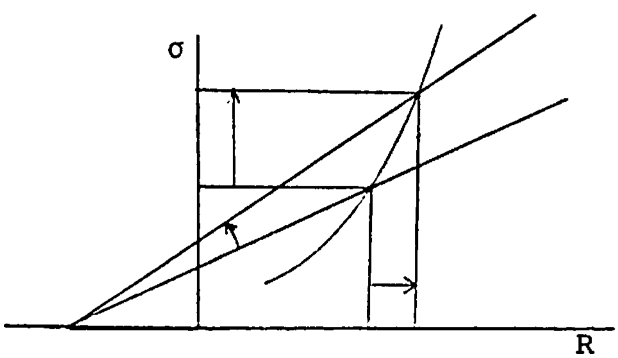
APPENDIX III--Continued



(9) increase in surplus



(10) decrease in k



(11) decrease in Z

APPENDIX III--Continued

Table III-1

Parameter Change(s)	Change in R	Change in σ	Change in X_1	Change in X_2
Shifts in Efficiency Frontier				
(1) Increase in r_1	+	+	-	+
(2) Increase in r_2	+	+	-	+
(3) Decrease in σ_{11}	+	+	-	+
(4) Decrease in σ_{22}	+	+	-	+
(5) Decrease in σ_{12}	+	+	-	+
(6) Yields increase by constant	+	+	-	+
(7) Proportional decrease in square roots of elements of variance-covariance matrix	+	+	-	+
(8) Proportional increase in elements of return vector and square roots of elements of variance-covariance matrix	+	+	+	-
Shifts in Preference Function				
(9) Increase in surplus	+	+	-	+
(10) Decrease in k	+	+	-	+
(11) Decrease in Z	+	+	-	+

All of these situations were discussed in Chapter III. Of course, parameter changes in the opposite direction of those listed above would cause opposite changes in R, σ, X_1, and X_2. The portfolio can be expanded to include more than two securities and any of the portfolio shifts above is possible in an n-security case as well.

APPENDIX IV-1

92 U.S. COMPANIES IN SAMPLE

	Name	State Charter	Stock/ Mutual	N.Y. License	12/31/71 Total Assets (in $ million)
1	The Prudential Insurance Co. of America	N.J.	M	yes	31,159.60
2	Metropolitan LIC	N.Y.	M	yes	29,163.29
3	The Equitable Life Assur. Soc. of the U.S.	N.Y.	M	yes	15,395.25
4	New York LIC	N.Y.	M	yes	11,268.29
5	John Hancock Mutual LIC	Mass.	M	yes	10,603.73
6	Aetna LIC	Conn.	S	yes	7,803.60
7	The Northwestern Mutual LIC	Wis.	M	yes	6,453.47
8	Connecticut General LIC	Conn.	S	yes	5,668.82
9	The Travelers Insurance Co.	Conn.	S	yes	5,043.96
10	Massachusetts Mutual LIC	Mass.	M	yes	4,566.07
11	The Mutual LIC of New York	N.Y.	M	yes	3,946.64
12	New England Mutual LIC	Mass.	M	yes	3,751.75
13	Connecticut Mutual LIC	Conn.	M	yes	2,922.38
14	Mutual Benefit LIC	N.J.	M	yes	2,698.00
15	Teachers Ins. and Annuity Assn. of America	N.Y.	S	yes	2,596.92
16	The Penn Mutual LIC	Pa.	M	yes	2,512.20
17	The Lincoln National LIC	Ind.	S	no	2,363.64
18	Bankers Life Co.	Iowa	M	yes	2,250.75
19	The Western and Southern LIC	Ohio	M	no	1,979.94
20	The National Life & Accident Insurance Co.	Tenn.	S	no	1,890.47
21	Occidental LIC of California	Calif.	S	no	1,796.04
22	Continental Assurance Co.	Ill.	S	yes	1,769.56
23	National LIC	Vt.	M	yes	1,521.94
24	American National Insurance Co.	Tex.	S	no	1,487.37

APPENDIX IV-1--<u>Continued</u>

	Name	State Charter	Stock/ Mutual	N.Y. License	12/31/71 Total Assets (in $ million)
25	Phoenix Mutual LIC	Conn.	M	yes	1,446.62
26	Franklin LIC	Ill.	S	no	1,311.25
27	State Mutual Life Assurance Co. of America	Mass.	M	yes	1,305.42
28	Provident Mutual LIC of Philadelphia	Pa.	M	yes	1,190.96
29	Southwestern LIC	Tex.	S	no	1,122.48
30	Jefferson Standard LIC	N.C.	S	no	1,056.13
31	The Guardian LIC of America	N.Y.	M	yes	993.12
32	Pacific Mutual LIC	Calif.	M	no	992.24
33	Equitable LIC of Iowa	Iowa	S	yes	958.06
34	The Union Central LIC	Ohio	M	yes	919.14
35	State Farm LIC	Ill.	S	no	902.20
36	The LIC of Virginia	Va.	S	no	894.79
37	Home LIC	N.Y.	M	yes	884.52
38	Liberty National LIC	Ala.	S	no	864.93
39	United Benefit LIC	Nebr.	S	no	770.03
40	Provident Life and Accident Insurance Co.	Tenn.	S	no	717.98
41	The Minnesota Mutual LIC	Minn.	M	no	694.80
42	Northwestern National LIC	Minn.	S	no	690.17
43	General American LIC	Mo.	M	no	607.13
44	Acacia Mutual LIC	Wash., D.C.	M	no	596.17
45	The Fidelity Mutual LIC	Pa.	M	yes	595.54
46	Nationwide LIC	Ohio	S	yes	577.04
47	Washington National Insurance Co.	Ill.	S	no	569.38
48	Bankers Life and Casualty	Ill.	S	no	567.20

APPENDIX IV-1--Continued

Name	State Charter	Stock/ Mutual	N.Y. License	12/31/71 Total Assets (in $ million)
49 Kansas City LIC	Mo.	S	no	555.20
50 Southland LIC	Tex.	S	no	553.11
51 Life and Casualty Insurance Co. of Tenn.	Tenn.	S	no	538.86
52 Commonwealth LIC	Ky.	S	no	506.33
53 Country LIC	Ill.	S	no	489.60
54 LIC of Georgia	Ga.	S	no	480.25
55 Pilot LIC	N.C.	S	no	472.19
56 California-Western States LIC	Calif.	S	no	458.55
57 United Insurance Co. of America	Ill.	S	no	456.49
58 The Paul Revere LIC	Mass.	S	yes	445.77
59 Monumental LIC	Md.	S	no	417.46
60 Gulf LIC	Fla.	S	no	414.62
61 Republic National LIC	Tex.	S	no	411.51
62 American United LIC	Ind.	M	no	405.43
63 Business Men's Assurance Co. of America	Mo.	S	no	396.33
64 Ohio National LIC	Ohio	M	no	394.19
65 Pan-American LIC	La.	M	no	394.04
66 Monarch LIC	Mass.	S	yes	373.23
67 Bankers LIC of Nebraska	Nebr.	M	no	370.46
68 Great Southern LIC	Tex.	S	no	363.48
69 Home Beneficial LIC	Va.	S	no	348.40
70 The Manhattan LIC	N.Y.	M	yes	344.32
71 Union Mutual LIC	Maine	M	yes	336.89
72 Central Life Assurance Co.	Iowa	M	no	318.12

APPENDIX IV-1--Continued

Name	State Charter	Stock/ Mutual	N.Y. License	12/31/71 Total Assets (in $ million)
73 Lutheran Mutual LIC	Iowa	M	yes	314.45
74 Berkshire LIC	Mass.	M	yes	311.04
75 Mutual Trust LIC	Ill.	M	yes	303.08
76 Liberty LIC	S.C.	S	no	302.10
77 The Independent Life and Accident Ins. Co.	Fla.	S	no	290.20
78 The United States LIC in the City of N.Y.	N.Y.	S	yes	283.33
79 Peoples LIC, Washington, D.C.	Wash., D.C.	S	no	279.78
80 Mass. Savings Bk. LI, Commonwealth of Mass.	Mass.	M	no	271.54
81 The Columbus Mutual LIC	Ohio	S	no	270.41
82 Indianapolis LIC	Ind.	M	no	263.34
83 Hartford LIC	Mass.	S	yes	252.86
84 Sun LIC of America	Md.	S	no	227.06
85 Guarantee Mutual Life Co.	Nebr.	M	no	224.35
86 Midland Mutual LIC	Ohio	M	no	206.26
87 Western LIC	Minn.	S	no	201.26
88 Equitable LIC	Wash., D.C.	S	no	200.35
89 Ohio State LIC	Ohio	S	no	200.10
90 Beneficial LIC	Utah	S	no	197.40
91 Continental American LIC	Del.	S	yes	192.23
92 The Colonial LIC of America	N.J.	S	yes	191.48

APPENDIX IV-2

12 CANADIAN COMPANIES IN SAMPLE

	Name	Stock/ Mutual	N.Y. License	12/31/71 Total Assets (in $ million)
1	Sun Life Assurance Co. of Canada	M	no	3,874.40
2	The Manufacturers LIC	M	no	2,192.60
3	London LIC	S	no	1,734.26
4	The Great-West Life Assurance Co.	S	no	1,634.46
5	The Canada Life Assurance Co.	M	yes	1,406.78
6	The Mutual Life Assurance Co. of Canada	M	no	1,288.08
7	Confederation Life Association	M	yes	933.89
8	Crown LIC	S	no	809.60
9	North American Life Assurance Co.	M	no	749.36
10	The Imperial Life Assurance Co. of Canada	S	no	533.42
11	The Dominion Life Assurance Co.	S	no	378.11
12	The Excelsior LIC	S	no	278.88

APPENDIX IV-3

26 COMPANIES EXCLUDED FROM SAMPLE

Name	State Charter	Stock/ Mutual	N.Y. License	12/31/71 Total Assets (in $ million)	Reason for Exclusion
Aid Association for Lutherans	Wis.	M	yes	937.78	inc. data[a]
Lutheran Brotherhood	Minn.	M.	yes	651.29	inc. data
Mutual of Omaha Insurance Co.	Nebr.	M	yes	625.09	inc. data
Knights of Columbus	Conn.	M	yes	441.38	inc. data
Woodmen of the World LI Society	Nebr.	M	yes	401.77	inc. data
American General LIC of Delaware	Del.	S	no	314.21	inc. data
Modern Woodmen of America	Ill.	M	yes	299.39	inc. data
Transamerica Ins. Corp. of Calif.	Calif.	S	no	298.79	inc. data
LIC of North America	Pa.	S	no	295.68	small size[b]
Southern Farm Bureau LIC	Miss.	S	no	290.98	small size
Allstate LIC	Ill.	S	yes	277.03	small size
Combined Ins. Co. of America	Ill.	S	no	276.18	inc. data
American General LIC	Tex.	S	no	256.77	small size
Royal Neighbors of America	Ill.	M	yes	224.83	inc. data
Fireman's Fund American LIC	Calif.	S	no	220.14	inc. data
New York Savings Bank Life Ins.	N.Y.	M	yes	220.00	small size

APPENDIX IV-3--Continued

Name	State Charter	Stock/ Mutual	N.Y. License	12/31/71 Total Assets (in $ million)	Reason for Exclusion
Fidelity Union LIC	Tex.	S	no	219.25	small size
American-Amicable LIC	Ala.	S	no	198.87	inc. data
Kentucky Central LIC	Ky.	S	no	198.72	small size
Interstate Life & Accident Ins. Co.	Tenn.	S	no	197.33	small size
American Health and LIC	Md.	S	no	195.92	small size
Standard Insurance Co.	Oreg.	M	no	193.42	small size
Philadephia LIC	Pa.	S	no	192.05	small size
The Independent Order of Foresters	Canada	M	yes	342.92	inc. data
The Nat. Life Assurance Co. of Canada	Canada	S	yes	267.50	small size
Industrial LIC	Canada	S	no	251.67	small size

[a]The company does not provide complete data back to 1957.

[b]The company possesses complete information, but its initial size may be insufficient for efficient diversification across all capital markets.

APPENDIX IV-4

CROSS-SECTIONAL/TIME-SERIES RESULTS FOR 104 COMPANIES, 1957-1971, USING YIELD LEVEL PROXY

	Dependent Variables					
Independent Variables	$\frac{B}{TA}$	$\frac{M}{TA}$	$\frac{F}{TA}$	$\frac{CS}{TA}$	$\frac{PS}{TA}$	$\frac{TS}{TA}$
CONSTANT	.489802	.492423	.982225	-.030729	-.011922	-.040979
$\frac{R}{TA}$	-.205647 (-4.99)	-.542003 (-11.97)	-.747650 (-23.85)	.465612 (31.73)	.168020 (20.50)	.630000 (36.99)
YC	-.014395 (-9.90)	-.006025 (-3.78)	-.020419 (-18.49)	.003653 (7.07)	.001739 (6.02)	.005290 (8.81)
SD	-.008555 (-1.70)	.042908 (7.79)	.034353 (9.00)	-.013229 (-7.41)	-.002876 (-2.88)	-.016106 (-7.77)
NL	.044059 (9.32)	-.050984 (-9.83)	-.006925 (-1.93)	.007750 (4.61)	.004831 (5.14)	.011817 (6.05)
NC	.005353 (.69)	.018368 (2.17)	.023721 (4.04)	-.017374 (-6.32)	.000614 (.40)	-.016849 (-5.28)
Std. Error of Estimate	.0790	.0867	.0601	.0281	.0157	.0326
R^2	.1762	.1532	.3894	.4458	.2559	.5202
$\bar{R}^2$	.1736	.1505	.3875	.4440	.2535	.5187
F-Ratio	66.50	56.23	198.23	250.03	106.88	337.02

APPENDIX IV-5

CROSS-SECTIONAL/TIME-SERIES RESULTS FOR 92 COMPANIES, 1957-1971, USING YIELD DIFFERENCES

	Dependent Variables					
Independent Variables	$\frac{B}{TA}$	$\frac{M}{TA}$	$\frac{B}{TA}$	$\frac{M}{TA}$	$\frac{PS}{TA}$	$\frac{PS}{TA}$
CONSTANT	.3792	.4558	.4183	.4521	-.006118	-.006535
(YM-YC)	.03027 (8.25)	.01684 (4.24)	-.001074 (-.10)	.01982 (1.64)	- -	.003443 (.15)
(YP-YC)	-	-	.03843 (2.98)	-.003659 (-.26)	-.006230 (-7.25)	-.006606 (-2.52)
$\frac{R}{TA}$	-.2320 (-5.33)	-.5405 (-11.46)	-.2234 (-5.13)	-.5143 (-11.45)	.171327 (19.42)	.1713 (19.38)
SD	.001525 (.28)	.03393 (5.78)	.0009769 (.18)	.03398 (5.79)	-.002247 (-2.05)	-.002242 (-2.04)
NL	.05021 (10.12)	-.05775 (-10.74)	.05032 (10.17)	-.05776 (-10.74)	.005913 (5.89)	.005913 (5.88)
NC	.005104 (.66)	.01630 (1.95)	.005190 (.67)	.01629 (1.94)	.000699 (.45)	.0006978 (.45)
Std. Error of Est.	.0784	.0849	.0782	.0849	.0159	.0159
R^2	.1825	.1590	.1878	.1591	.2793	.2793
$\bar{R}^2$	.1795	.1560	.1842	.1554	.2767	.2761
F-Ratio	61.35	51.96	52.90	43.29	106.48	88.68

APPENDIX IV-6

OLS ESTIMATES USING DIFFERENTIAL SLOPES AND INTERCEPTS FOR CANADIAN FIRMS

	Dependent Variables					
Independent Variables	$\frac{B}{TA}$	$\frac{M}{TA}$	$\frac{F}{TA}$	$\frac{CS}{TA}$	$\frac{PS}{TA}$	$\frac{TS}{TA}$
CONSTANT	.4760	.5102	.9862	-.0369	-.0152	-.0510
$\frac{R}{TA}$	-.1966 (-4.64)	-.5804 (-12.49)	-.7770 (-24.01)	.5019 (34.12)	.1748 (20.70	.6744 (39.47)
YC	-.0123 (-8.02)	-.0076 (-4.57)	-.0200 (-17.06	.0033 (6.20)	.0021 (6.84)	.0053 (8.65)
CD	.0208 (.58)	-.0235 (-.60)	-.0027 (-.10)	-.0007 (-.06)	.0228 (3.21)	.0292 (2.03)
$C\frac{R}{TA}$	1.5035 (3.59)	-1.4205 (-3.09)	.0831 (.26)	.1578 (1.09)	-.0381 (-.46)	.0559 (.33)
CYC	-.0245 (-5.10)	.0204 (3.88)	-.0041 (-1.11)	.0020 (1.23)	-.0030 (-3.09)	-.0011 (-.60)
SD	-.0092 (-1.85)	.0437 (8.01)	.0345 (9.07)	-.0135 (-7.81)	-.0030 (-2.98)	-.0165 (-8.19)
NL	.0454 (9.52)	-.0548 (-10.48)	-.0094 (-2.59)	.0110 (6.62)	.0053 (5.59)	.0156 (8.10)
NC	.0055 (.72)	.0165 (1.96)	.0221 (3.76)	-.0154 (-5.75)	.0010 (.62)	-.0144 (-4.65)
Std. Error of Estimate	.0783	.0859	.0598	.0272	.0156	.0316

APPENDIX IV-6 Continued

OLS ESTIMATES USING DIFFERENTIAL SLOPES AND INTERCEPTS FOR CANADIAN FIRMS

	Dependent Variables					
Independent Variables	$\frac{B}{TA}$	$\frac{M}{TA}$	$\frac{F}{TA}$	$\frac{CS}{TA}$	$\frac{PS}{TA}$	$\frac{TS}{TA}$
R^2	.1026	.1704	.3951	.4822	.2650	.5511
$\bar{R}^2$	.1884	.1661	.3920	.4795	.2612	.5488
F-Ratio	46.24	39.81	126.63	180.52	69.91	238.06

APPENDIX IV-7

CROSS-SECTIONAL/TIME-SERIES REGRESSIONS INCORPORATING AN ADJUSTMENT FOR FIRST ORDER AUTOCORRELATION

	Dependent Variable is $\frac{B}{TA} - r(\frac{B}{TA})_{lagged}$				
Independent Variables	r = 0.0	r = 0.5	r = 0.8	r = 0.9	r = 1.0
CONSTANT	.470193	.229478	.086889	.0418598	-.001030
$\frac{R}{TA} - r(\frac{R}{TA})_{lagged}$	-.241776 (-5.48)	-.231734 (-5.43)	-.203180 (-5.06)	-.1885795 (-4.73)	-.197246 (-4.77)
$YC - rYC_{lagged}$	-.011834 (-7.61)	-.009880 (-7.28)	-.006328 (-5.86)	-.0047583 (-4.67)	-.003951 (-3.64)
SD	.003573 (.65)	.002394 (.85)	.001672 (1.20)	.0016299 (1.48)	.001870 (1.72)
NL	.047764 (9.44)	.021074 (8.11)	.005079 (3.75)	-.0002855 (-.25)	-.005705 (-5.15)
NC	.005616 (.71)	.003751 (.93)	.002608 (1.23)	.0021877 (1.25)	.001757 (1.01)
Std. Error of Est.	.0772	.0397	.0207	.0172	.0170
R^2	.1742	.1459	.0650	.0288	.0503
$\bar{R}^2$	.1710	.1426	.0613	.0250	.0466
F-Ratio	54.08	43.80	17.82	7.59	13.58

APPENDIX IV-7--Continued

Independent Variables	Dependent Variable is $\frac{M}{TA} - r(\frac{M}{TA})_{lagged}$				
	r = 0.0	r = 0.5	r = 0.8	r = 0.9	r = 1.0
CONSTANT	.513580	.258231	.099265	.045962	-.004361
$\frac{R}{TA} - r(\frac{R}{TA})_{lagged}$	-.549811 (-11.44)	-.537370 (-11.52)	-.479127 (-10.96)	-.420389 (-9.85)	-.3640398 (-8.45)
$YC - rYC_{lagged}$	-.007835 (-4.62)	-.009021 (-6.08)	-.007966 (-6.78)	-.005865 (-5.37)	-.0026920 (-2.38)
SD	.034414 (5.71)	.017283 (5.61)	.006664 (4.39)	.003304 (2.79)	.0006835 (.63)
NL	-.056895 (-10.32)	-.027714 (-9.75)	-.010110 (-6.85)	-.004247 (-3.53)	.0014815 (1.28)
NC	.017735 (2.06)	.010414 (2.35)	.006021 (2.62)	.004480 (2.38)	.0028160 (1.56)
Std. Error of Est.	.0841	.0434	.0225	.0184	.0177
R^2	.1625	.1615	.1257	.0845	.0580
$\bar{R}^2$	.1592	.1582	.1223	.0809	.0543
F-Ratio	49.75	49.39	36.87	23.66	15.59

APPENDIX IV-7--Continued

Independent Variables	Dependent Variable is $\frac{F}{TA} - r(\frac{F}{TA})_{lagged}$				
	r = 0.0	r = 0.5	r = 0.8	r = 0.9	r = 1.0
CONSTANT	.983772	.487709	.186154	.087822	-.005366
$\frac{R}{TA} - r(\frac{R}{TA})_{lagged}$	-.791587 (-22.14)	-.769103 (-20.93)	-.682307 (-17.46)	-.608968 (-15.35)	.561286 (-14.51)
$YC - rYC_{lagged}$	-.019668 (-15.59)	-.018901 (-16.17)	-.014295 (-13.62)	-.010623 (-10.46)	-.006643 (-6.55)
SD	.037987 (8.46)	.019678 (8.11)	.008336 (6.14)	.004934 (4.48)	.002491 (2.54)
NL	-.009131 (-2.23)	-.006640 (-2.97)	-.005031 (-3.82)	-.004532 (-4.05)	-.004223 (-4.08)
NC	.023351 (3.65)	.014166 (4.06)	.008629 (4.20)	.006668 (3.82)	.004573 (2.82)
Std. Error of Est.	.0626	.0342	.0201	.0171	.0159
R^2	.3816	.3576	.2586	.1923	.1588
$\bar{R}^2$	.3792	.3551	.2557	.1892	.1555
F-Ratio	158.20	142.71	89.43	61.05	48.39

APPENDIX IV-7--Continued

	Dependent Variable is $\frac{CS}{TA} - r(\frac{CS}{TA})_{lagged}$				
Independent Variables	r = 0.0	r = 0.5	r = 0.8	r = 0.9	r = 1.0
CONSTANT	-.036747	-.017663	-.006143	-.0022946	.0016447
$\frac{R}{TA} - r(\frac{R}{TA})_{lagged}$	.511343 (31.68)	.533990 (30.71)	.610680 (28.31)	.6571892 (27.35)	.6761281 (26.80)
$YC - rYC_{lagged}$	.003049 (5.35)	.002581 (4.67)	.001220 (2.11)	.0004609 (.75)	-.0004889 (-.74)
SD	-.013667 (-6.75)	-.007707 (-6.71)	-.004338 (-5.79)	-.0027386 (-4.11)	-.0005129 (-.80)
NL	.012303 (6.64)	.006527 (6.16)	.003142 (4.32)	.0019428 (2.87)	.0006120 (.91)
NC	-.016071 (-5.57)	-.008106 (-4.91)	-.003366 (-2.97)	-.0018912 (-1.79)	-.0004911 (-.46)
Std. Error of Est.	.0283	.0162	.0111	.0104	.0104
R^2	.4922	.4661	.4044	.3844	.3785
$\bar{R}^2$	.4902	.4640	.4020	.3820	.3761
F-Ratio	248.53	223.81	174.05	160.13	156.15

APPENDIX IV-7--Continued

Independent Variables	r = 0.0	r = 0.5	r = 0.8	r = 0.9	r = 1.0
	Dependent Variable is $\frac{PS}{TA} - r(\frac{PS}{TA})_{lagged}$				
CONSTANT	-.0158882	-.007184264	-.0009129	.0005874	.0003664
$\frac{R}{TA} - r(\frac{R}{TA})_{lagged}$	.1775862 (19.21)	.170788701 (17.27)	.1176418 (10.37)	.0674097 (5.71)	.0261417 (2.28)
$YC - rYC_{lagged}$	.0020359 (6.24)	.001816931 (5.77)	.0009304 (3.05)	.0001656 (.55)	-.0005929 (-1.97)
SD	-.0021574 (-1.86)	-.000345510 (-.53)	.0012284 (3.12)	.0015631 (4.77)	.0012543 (4.31)
NL	.0060298 (5.69)	.002974926 (4.94)	.0010211 (2.67)	.0003816 (1.15)	-.0001362 (-.44)
NC	.0005893 (.36)	.000002847 (.003)	-.0003746 (-.63)	-.0004301 (-.83)	-.0003864 (-.80)
Std. Error of Est.	.0162	.0092	.0058	.0051	.0047
R^2	.2801	.2440	.1230	.0625	.0314
$\bar{R}^2$	.2773	.2410	.1196	.0589	.0276
F-Ratio	99.77	82.74	35.96	17.10	8.31

APPENDIX IV-7--Continued

	Dependent Variable is $\frac{TS}{TA} - r(\frac{TS}{TA})_{lagged}$				
Independent Variables	r = 0.0	r = 0.5	r = 0.8	r = 0.9	r = 1.0
CONSTANT	-.051513	-.024358	-.006942	-.0016870	.0019787
$\frac{R}{TA} - r(\frac{R}{TA})_{lagged}$	.688702 (36.73)	.704686 (35.01)	.728930 (30.16)	.7257942 (27.29)	.7037739 (25.48)
$YC - rYC_{lagged}$	.005044 (7.62)	.004380 (6.88)	.002182 (3.36)	.0006843 (1.01)	-.0010043 (-1.39)
SD	-.016320 (-6.93)	-.008321 (-6.30)	-.003247 (-3.87)	-.0012659 (-1.72)	.0007057 (1.01)
NL	.017623 (8.19)	.009162 (7.51)	.004047 (4.96)	.0022817 (3.04)	.0005056 (.68)
NC	-.015584 (-4.65)	-.008158 (-4.30)	-.003767 (-2.97)	-.0023396 (-2.00)	-.0008883 (-.77)
Std. Error of Est.	.0328	.0186	.0125	.0115	.0114
R^2	.5673	.5396	.4445	.3915	.3612
$\bar{R}^2$	.5656	.5378	.4424	.3891	.3587
F-Ratio	3336.19	300.48	205.20	164.93	144.97

BIBLIOGRAPHY

SELECTED BIBLIOGRAPHY

Books

Best's Insurance Reports. Life-Health Edition. Editions 53-67. Park Avenue, Morristown, New Jersey: A.M. Best Company, Inc., 1958-1972.

Borch, Karl Henrik. The Economics of Uncertainty. Princeton, New Jersey: Princeton University Press, 1968.

Brimmer, Andrew F. Life Insurance Companies in the Capital Market. East Lansing: Michigan State University Press, 1962.

Cissell, Robert, and Cissell, Helen. Mathematics of Finance. 3rd ed. Boston: Houghton-Mifflin Company, 1969.

Clayton, G. and Osborn, W.T. Insurance Company Investment, Principals and Policies. London: George Allen and Unwin, Ltd., 1965.

Eckstein, Otto. "A Survey in the Theory of Public Expenditure Criteria, Reply." In Conference of the Universities--National Bureau Committee for Economic Research. Public Finance: Needs, Sources, and Utilization. Princeton: Princeton University Press, 1961.

Espie, Robert G. "Financial Statements." Life Insurance. revised edition by Dan M. McGill. Homewood, Illinois: Richard D. Irwin, Inc., 1967.

Fraine, Harold G. Valuation of Securities Holdings of Life Insurance Companies. Homewood, Illinois: Richard D. Irwin, Inc., 1962.

Francis, Jack, and Archer, Stephen. Portfolio Analysis. Englewood Cliffs, New Jersey: Prentice Hall, Inc., 1971.

Goldberger, A.S. Econometric Theory. New York: John Wiley and Sons, Inc., 1964.

Harper, Floyd S., and Workman, Lewis C. Fundamental Mathematics of Life Insurance. Homewood, Illinois: Richard D. Irwin, Inc., 1970.

Hickman, W. Braddock. Corporate Bond Quality and Investor Experience. New York: National Bureau of Economic Research, 1958.

Institute of Life Insurance. 1971 Life Insurance Fact Book. New York: Institute of Life Insurance, 1971.

Institute of Life Insurance. 1972 Life Insurance Fact Book. New York: Institute of Life Insurance, 1972.

Johnston, J. Econometric Methods. 2nd ed. New York: McGraw-Hill, 1972.

Jones, Lawrence D. Investment Policies of Life Insurance Companies. Boston: Division of Research, Graduate School of Business Administration, Harvard University, 1968.

Klein, Jack G. "The Liquidity Structure of Life Insurance Companies." Unpublished Ph.D. dissertation, University of Pennsylvania, 1965.

Markowitz, Harry. Portfolio Selection: Efficient Diversification of Investments. New York: John Wiley and Sons, Inc., 1959.

McGill, Dan M. Life Insurance. Rev. ed. Homewood, Illinois: Richard D. Irwin, Inc., 1967.

Moore, Basil J. An Introduction to the Theory of Finance: Assetholder Behavior Under Uncertainty. New York: The Free Press, 1968.

Olson, Alden C. The Impact of Valuation Requirements on the Preferred Stock Investment Policies of Life Insurance Companies; Occasional Paper No. 13. East Lansing, Michigan: Bureau of Business and Economic Research, Graduate School of Business Administration, Michigan State University, 1964.

Silber, William L. Portfolio Behavior of Financial Institutions: An Empirical Study with Implications for Monetary Policy, Interest-Rate Determination, and Financial Model-Building. New York: Holt, Rinehart and Winston, 1970.

Walter, James E. The Investment Process As Characterized by Leading Life Insurance Companies. Boston: Harvard University Press, 1962.

Articles

Abelle, Bernie E., Jr. "An Evaluation of the Life Insurance Company Income Tax Act of 1959." Journal of Insurance, XXX (September, 1963), 403-22.

Agnew, N.H.; Agnew, R.A.; Rasmussen, J.; and Smith, K.R. "An Application of Chance Constrained Programming to Portfolio Selection in a Casualty Insurance Firm." Management Science, XV (June, 1969), B512-20.

Almon, Shirley. "The Distributed Lag Between Capital Appropriations and Expenditures." Econometrica, XXXIII (January, 1965), 178-96.

Anderson, W.M. "The Long View of Life Insurance Investment." American Life Convention General Proceedings, Chicago: American Life Convention, 1954.

Baumol, William J. "An Expected Gain-Confidence Limit Criterion for Portfolio Selection." Management Science, X (October, 1963), 174-82.

Brainard, William C., and Tobin, James. "Pitfalls in Financial Model Building." American Economic Review, LVIII (May, 1968), 99-122.

Charnes, A., and Thore, Sten. "Planning for Liquidity in Financial Institutions: The Chance-Constrained Method." Journal of Finance, XXI (December, 1966), 649-74.

Chow, Gregory C. "Tests of Equality Between Sets of Coefficients in Two Linear Regressions." Econometrica, XXVIII (July, 1960), 591-605.

Fisher, Lawrence. "Determinants of Risk Premiums on Corporate Bonds." Journal of Political Economy, LXVII (June, 1959), 217-37.

Fried, Joel. "Bank Portfolio Selection." Journal of Financial and Quantitative Analysis, V (June, 1970), 203-27.

Griliches, Zvi. "Distributed Lags: A Survey." Econometrica, XXXV (January, 1967), 16-49.

Hadar, Josef, and Russell, William R. "Rules for Ordering Uncertain Prospects." American Economic Review, LIX (March, 1969), 25-34.

Hakansson, Nils H. "Multi-period Mean-Variance Analysis: Toward a General Theory of Portfolio Choice." Journal of Finance, XXVI (September, 1971), 857-84.

Hirschlaefer, Jack. "Efficient Allocation of Capital in an Uncertain World." American Economic Review, LVI (May, 1964), 77-85.

Hildreth, Clifford, and Lu, John Y. "Demand Relations with Autocorrelated Disturbances." Michigan State University Technical Bulletin 276, (November, 1960), 1-76.

Jen, Frank C., and Wert, James E. "The Value of the Deferred Call Privilege." National Banking Review, III (March, 1966), 369-78.

Kaufman, Stephen M. "The Life Insurance Company Income Tax Act of 1959: An Appraisal of the Effects Upon the Life Insurance Industry, Part II." National Tax Journal, XVII (March, 1964), 40-56.

Lintner, John. "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets." Review of Economics and Statistics, XLVII (February, 1965), 13-37.

Maddala, G.S. "The Use of Variance Components Models in Pooling Cross Section and Time Series Data." Econometrica, XXXIX (March, 1971), 341-58.

Markowitz, Harry. "Portfolio Selection." Journal of Finance, VII (March, 1952), 77-91.

Mossin, Jan. "Optimal Multiperiod Portfolio Policies." The Journal of Business, XLI (April, 1968), 215-29.

Nerlove, Marc. "Factors Affecting Differences Among Rates of Return on Investments in Individual Common Stocks." Review of Economics and Statistics, L (August, 1968), 312-31.

O'Leary, James J. "The Institutional Saving-Investment Process and Current Economic Theory." American Economic Review, XLIV (May 1954), 455-70.

Schott, Frances H. "Disintermediation Through Policy Loans at Life Insurance Companies." Journal of Finance, XXVI (June, 1971), 719-29.

Sharpe, William F. "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk." Journal of Finance, XIX (September, 1964), 425-42.

Silber, William L. "Portfolio Substitutability, Regulations, and Monetary Policy." Quarterly Journal of Economics, LXXXIII (May, 1969), 196-219.

Wehrle, Leroy S. "Life Insurance Investment--the Experience of Four Companies." Yale Economic Essays, I (Spring, 1961), 70-136.

Whitman, Andrew F., and Thompson, Howard E. "The Impact of the 1959 Income Tax Act on Stock and Mutual Companies: A Simulation Study." Journal of Risk and Insurance, XXXIV (June, 1967), 215-229.